DEPARTMENT OF CIVIL ENGINEERING

(ACADEMIC YEAR 2025 -26)

SUBJECT NAME: ANALYSIS OF STRUCTURES

SUB CODE: BCV401

SEMESTER: IV

ATME College of Engineering

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INSTITUTE

Vision of the Institute

Development of academically excellent, culturally vibrant, socially responsible and globally competent human resources.

Mission of the Institute

- To keep pace with advancements in knowledge and make the students competitive and capable at the global level.
- To create an environment for the students to acquire the right physical, intellectual, emotional and moral foundations and shine as torch bearers of tomorrow's society.
- To strive to attain ever-higher benchmarks of educational excellence

DEPARTMENT

Vision of the Department

To develop globally competent Civil Engineers who excel in academics, research and are ethically responsible for the development of the society.

Mission of the Department

- To provide quality education through faculty and state of art infrastructure
- To identify the current problems in society pertaining to Civil Engineering disciplines and to address them effectively and efficiently
- To inculcate the habit of research and entrepreneurship in our graduates to address current infrastructure needs of society

Program outcomes (POs)

Engineering Graduates will be able to:

PO1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

PO2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

PO3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

PO4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

PO6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

PO10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO12. Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change

Program Specific Outcomes (PSOs)

PSO1: Provide necessary solutions to build infrastructure for all situations through Competitive plans, maps and designs with the aid of a thorough Engineering Survey and Quantity Estimation.

PSO2: Assess the impact of anthropogenic activities leading to environmental imbalance on land, in water & in air and provide necessary viable solutions revamping water resources and transportation for a sustainable development.

Program Educational Objectives (PEOs)

PEO 1- Engaged in professional practices, such as construction, environmental, geotechnical, structural, transportation, water resource engineering by using technical, communication and management skills.

PEO 2- Engaged in higher studies and research activities in various civil engineering fields and life time commitment to learn ever changing technologies to satisfy increasing demand of sustainable infrastructural facilities.

PEO 3- Serve in a leadership position in any professional or community organization or local or state engineering board

PEO 4- Registered as professional engineer or developed a strong ability leading to professional licensure being an entrepreneur.

MODULE 1

Introduction and Analysis of Plane Trusses

1. Structural Elements

There are **three types of structural elements**:

(i) Beams

- Horizontal members subjected to **transverse loads**
    - Point Load (PL)
    - Uniformly Distributed Load (UDL)
    - Uniformly Varying Load (UVL)

Based on support system, beams are classified as:

1. Cantilever Beam
2. Simply Supported Beam
3. Continuous Beam
4. Propped Cantilever
5. Overhanging Beam

(ii) Columns

- Vertical compression members.
- Transfer load to the substructure.

Based on loading condition:

1. Axial compression only
2. Axial load + uniaxial bending moment
3. Axial load + biaxial bending moment

(iii) Tie Members

- Horizontal members subjected to **axial tension**.
- Usually used in **trusses**.

2. Structural Forms

(i) Trusses

- Pin-jointed (hinged) structures.
- Members arranged in **triangular form**.
- Loads are applied only at the **joints**.
- Members carry **axial forces only** (tension or compression).

(ii) Frames

- Combination of beams and columns.
- Supported by fixed or hinged supports.
- Loads applied on beams.
- Members subjected to:
    - Shear Force (SF)
    - Axial Force
    - Bending Moment (BM)

(iii) Arches

- Curved members (parabolic or circular).
- Used for long-span bridges.
- Subjected mainly to **axial compression**.
- Types:
    - Three-hinged arch
    - Two-hinged arch

(iv) Cables

- Inverted curved members.
- Subjected to **axial tension**.
- Used in suspension bridges.

3. Equilibrium Conditions

Used for analysis of **statically determinate structures**.

[$\Sigma F_x = 0$]
[$\Sigma F_y = 0$]
[$\Sigma M = 0$]

4. Compatibility Conditions

- Used for analysis of **statically indeterminate structures**.
- Based on displacement compatibility.
- Structure must deform without separation or overlap.

5. Difference Between Statically Determinate and Indeterminate Structures

Statically Determinate	Statically Indeterminate
Solved using equilibrium equations only	Requires compatibility conditions
Independent of material properties	Depends on material properties
No stresses due to temperature change	Stresses induced due to temperature change
No stresses due to lack of fit	Stresses induced due to lack of fit

6. Support System

Types of Supports (2D and 3D)

Support Type	2D Reactions	3D Reactions
Fixed	3	6
Roller	1	1
Hinged	2	3

7. Internal Hinges and Links

1. Internal Hinge → Bending moment = 0
2. Internal Guided Roller
    - Reaction = 2

    - Axial force = 0
    - Shear force = 0

8. Indeterminacy of Structures

Types:

1. **Static Indeterminacy (Ds)** → Unknown internal forces
2. **Kinematic Indeterminacy (Dk)** → Unknown displacements

Determinate Structure

Number of unknowns = Number of equilibrium equations

For 2D:
$$E = 3$$

Indeterminate Structure

Number of unknowns > Number of equilibrium equations

9. Degree of Static Indeterminacy (Ds)

$$Ds = \text{Total unknowns} - \text{Total equilibrium equations}$$

Stability:

- If Ds < 0 → Unstable
- If Ds = 0 → Just stable
- If Ds > 0 → Stable and indeterminate

10. Static Indeterminacy for Beams

$$Ds = Dse + Dsi$$

External Static Indeterminacy:

$$Dse = R - E - A$$

Where:

- R = Number of unknown reactions
- E = Equilibrium equations (3 in 2D)

- A = Additional equations due to internal hinge

For internal hinge:
$[A = \sum (m' - 1)]$

11. Static Indeterminacy for Plane Trusses

$[D_s = (m + r) - 2j]$

Where:

- m = Number of members
- r = Number of reactions
- j = Number of joints

Condition for determinate truss:
$[m + r = 2j]$

12. Static Indeterminacy for Plane Frames

Each member → 3 unknown forces (Axial, Shear, Moment)

$[D_s = (3m + R) - (3j + p)]$

Where:

- m = Number of members
- R = Reactions
- j = Joints
- p = Additional equations due to hinge

13. Static Indeterminacy for Space Frames

Equilibrium equations in 3D:

$[\Sigma F_x = 0]$
$[\Sigma F_y = 0]$
$[\Sigma F_z = 0]$
$[\Sigma M_x = 0]$
$[\Sigma M_y = 0]$
$[\Sigma M_z = 0]$

Total equations = 6

External indeterminacy:
$$EI = R - 6$$

Internal indeterminacy:
$$II = m \times c$$

Releases:
$$R' = \sum (m' - 1)$$

$$Ds = EI + II - R'$$

14. Static Indeterminacy for Plane Frames (Alternative Expression)

$$SI = EI + II - R'$$

Where:

- EI = R - r (r = equilibrium equations)
- II = m × c (c = number of closed loops)
- R' = Σ (m' - 1)

15. Degree of Kinematic Indeterminacy (Dk)

Also called **Degree of Freedom (DOF)**

Definition:

Total number of independent joint displacements.

For Beams (Considering axial deformation):

$$Dk = 3j - R + A$$

Where:

- j = Number of joints
- R = External reactions
- A = Additional DOF due to internal hinge

DOF Concept

At any joint:

- Horizontal displacement

- Vertical displacement
- Rotation

16. Type of Structure vs Indeterminacy

Structure	Static Indeterminacy	Kinematic Indeterminacy
Beam (2D)	Dse = R - E	Dk = 3j - Re
Frame (2D)	Ds = (3m + R) - (3j + p)	Dk = 3j - Re
Truss (2D)	Ds = (m + R) - 2j	Dk = 2j - Re

For 3D:

- Dk = 6j - Re

17. Numerical Applications

The module includes several solved examples covering:

- Kinematic indeterminacy of beams:
    - Axially flexible
    - Axially rigid

Formulas used:
[Dk = 3j - Re]
[Dk = 3j - Re - m']

- Static and kinematic indeterminacy of:
    - Continuous beams
    - Multi-bay frames
    - Trusses
    - Rigid jointed frames

All solved cases strictly apply the formulas derived above.

Final Summary of Key Formulae

Beam:

[Dse = R - E - A]

Plane Truss:

[Ds = (m + r) - 2j]

Plane Frame:

[Ds = (3m + R) - (3j + p)]

Space Frame:

[Ds = EI + II - R']

Kinematic Indeterminacy:

2D: [Dk = 3j - Re]

3D: [Dk = 6j - Re]

MODULE 2

Deflection of Beams & Strain Energy Methods

DEFLECTION OF BEAMS

1. Introduction to Beam Deflection

When a beam is subjected to bending:

- Internal bending moment develops.
- Beam curvature is produced.
- Vertical displacement occurs.
- Rotation (slope) occurs at sections.

Understanding deflection is important for:

- Serviceability limit state
- Crack control
- Structural stiffness evaluation
- Indeterminate structure analysis

2. Basic Differential Equation of Elastic Curve

From bending theory:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

For small deflections:

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Therefore,

$$EI\frac{d^2y}{dx^2} = M(x)$$

.This is the **governing equation of beam deflection**.

Where:

- E = Modulus of Elasticity
- I = Moment of Inertia
- y = Deflection
- M(x) = Bending moment function

3. Moment Area Method

This method is based on integration of:

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

4. Derivation of Moment Area Theorems

Starting from:

$$EI\frac{d^2y}{dx^2} = M$$

Integrate once:

$$EI\frac{dy}{dx} = \int M dx$$

Since:

$$\frac{dy}{dx} = \theta$$

Then:

$$EI\theta = \int M dx$$

Thus:

$$\theta = \frac{1}{EI}\int M dx$$

Mohr's First Theorem

The change in slope between two points on a beam equals the area of the M/EI diagram between those two points.

$$\theta_B - \theta_A = \frac{1}{EI}\int_A^B M dx$$

Mohr's Second Theorem

The deviation of point B with respect to tangent drawn at point A equals the moment of area of M/EI diagram between A and B about point B.

$$\delta_{B/A} = \frac{1}{EI}\int_A^B Mx dx$$

5. Sign Convention (Very Important)

Bending Moment

- Sagging → Positive

- Hogging → Negative

Slope

- Clockwise → Positive
- Anticlockwise → Negative

Deflection

- Downward → Positive
- Upward → Negative

Consistency is essential.

6. Application to Determinate Prismatic Beams

For constant EI:

$$\theta = \frac{Area\ of\ BMD}{EI}$$

$$\delta = \frac{Moment\ of\ area\ of\ BMD}{EI}$$

Used for:

- Simply supported beams
- Cantilever beams
- Overhanging beams

7. Beams of Varying Cross Section

If EI is not constant:

$$\theta = \int \frac{M}{EI} dx$$

Here:

- Divide beam into segments.

- Use piecewise integration.
- Use moment diagram by parts.

8. Moment Diagram by Parts

Instead of full integration:

- Break BMD into geometric shapes.
- Compute area of each shape.
- Compute centroid of each shape.
- Apply Mohr's theorems.

Advantages:

- Fast
- Suitable for exams
- Graphical understanding

PART B

STRAIN ENERGY METHODS

9. Principle of Virtual Displacements

If a structure is in equilibrium, total virtual work done by external forces equals internal virtual work.

$$\delta W_{external} = \delta W_{internal}$$

Used in displacement method.

10. Principle of Virtual Forces

If structure is in equilibrium, work done by real forces through virtual displacements equals work done by virtual forces through real displacements.

Used in unit load method.

11. Strain Energy

Strain energy = Energy stored due to deformation.

$$U = \int \frac{\sigma\epsilon}{2} dV$$

11.1 Strain Energy due to Axial Force

$$U = \frac{P^2 L}{2AE}$$

11.2 Strain Energy due to Bending

$$U = \int \frac{M^2}{2EI} dx$$

11.3 Strain Energy due to Shear

$$U = \int \frac{V^2}{2GA} dx$$

11.4 Strain Energy due to Torsion

$$U = \int \frac{T^2}{2GJ} dx$$

12. Complementary Energy

Complementary energy is:

$$U^* = \int \sigma d\epsilon$$

For linear elastic materials:

$$U = U^*$$

13. Castigliano's First Theorem

Partial derivative of strain energy with respect to load gives displacement in direction of that load.

$$\delta = \frac{\partial U}{\partial P}$$

14. Castigliano's Second Theorem

Partial derivative of strain energy with respect to moment gives rotation.

$$\theta = \frac{\partial U}{\partial M}$$

15. Application to Beams

Procedure:

1. Express bending moment M(x).
2. Write strain energy:

$$U = \int \frac{M^2}{2EI} dx$$

3. Differentiate wrt load:

$$\delta = \frac{\partial U}{\partial P}$$

16. Application to Trusses

For truss member:

For truss member:

$$U = \sum \frac{F^2 L}{2AE}$$

Deflection:

$$\delta = \frac{\partial U}{\partial P}$$

17. Application to Frames

Include:

- Axial energy
- Bending energy

$$U = \int \frac{M^2}{2EI} dx + \int \frac{P^2}{2AE} dx$$

Then apply Castigliano theorem.

18. Comparison: Moment Area vs Castigliano

Moment Area	Castigliano
Geometric method	Energy method
Good for determinate beams	Works for determinate & indeterminate
Uses BMD	Uses M^2
Graphical	Analytical