Air Standard ~~Gas Power~~ Cycles

Gas power cycles are thermodynamics cycles, in which the working substances remains a gas throughout the cycle of operation.

In the thermodynamic analysis of power cycles, the chief interest lies in estimating the energy conversion efficiency (thermal efficiency) and how the major parameters (pressure, volume and temperature variations etc ...) of the cycles affect the performance of the heat engine. Such analysis are made based on the following assumptions:

(i) Air is used as the working substance, it behaves as perfect gas i.e it obeys the gas laws and has constant specific heats (C_p and C_v).

(ii) The engine operates in a closed cycle. The cylinder is filled with constant amount of air and the same air is used repeatedly.

(iii) No chemical reaction takes place in the engine cylinder. Heat is supplied (or) rejected by bringing a hot body (or) a cold body in contact with the cylinder head at appropriate time during the process.

(iv) Compression and expansion processes are adiabatic (insulated) and internally reversible (no mechanical (or) friction loss).

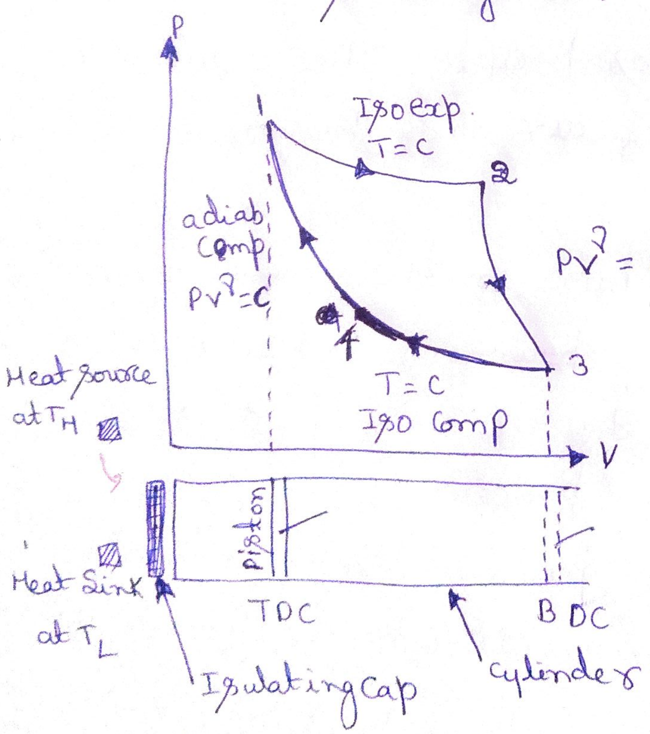
The efficiency calculated under the above discussed ideal conditions is known as ideal efficiency (or) air standard efficiency. However, under actual conditions variations occurs because air and fuel mixture is used as the working substance, intake and exhaust condition vary and also actual combustion process is different.

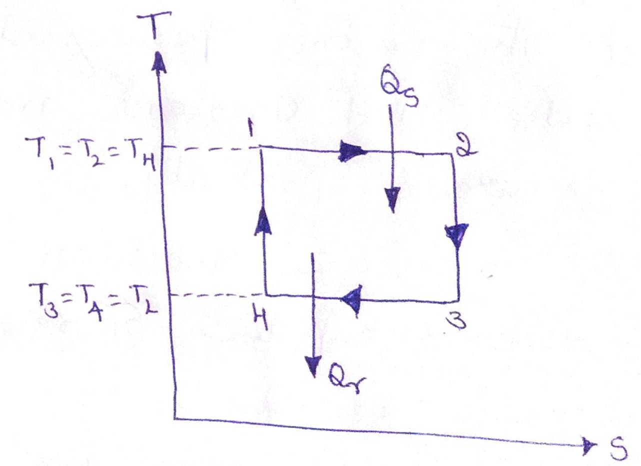
Also losses due to mechanical and friction effects need to be taken into account. Thus the actual efficiency of the cycle is always less than the air standard efficiency. and this is measured by a term known as relative efficiency.

$$\eta_{rel} = \frac{\text{actual thermal efficiency}}{\text{air standard efficiency}}$$

Carnot Cycle :

Carnot cycle consists of four process as shown in p-v and t-s diagram.





Let the cylinder contain 'm' kg of air at its initial condition represented by point 1 on p-v and t-s diagrams. The various processes involved in Carnot cycle are discussed below:

Process 1-2 Isothermal expansion:

The air in the cylinder is heated by bringing the hot body in contact with the cylinder head. The heat supplied by the hot body at constant temperature T_1 is fully absorbed by the air in the cylinder and this heat is utilized for doing external work (piston movement).

∴ Heat absorbed (or) heat added (or) heat supplied = Q_s

$$Q_s = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

under ideal conditions, we have $PV = mRT$ i.e $P_1 V_1 = mRT_1$

$$\therefore Q_s = mRT_1 \ln\left(\frac{V_2}{V_1}\right) \longrightarrow (1)$$

Process 2-3 Adiabatic Expansion:

∴ The heat source (hot body) is removed and an insulating cap is brought in contact with the cylinder head. Air expands adiabatically and the temperature falls from T_2 to T_3.

W.K.T from adiabatic process,

heat transfer $Q = 0$ and $\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \longrightarrow (2)$

Process 3-4 Isothermal Compression:

The insulating cap is removed and a cold body is brought in contact with the cylinder head. The piston moves upwards compressing the air at constant temperature T_3. The heat rejected at constant temperature T_3 is given by:

$$\text{Heat rejected} = Q_r = P_3 V_3 \ln\left(\frac{V_3}{V_4}\right)$$

$$= mRT_3 \ln\left(\frac{V_3}{V_4}\right) \longrightarrow (3)$$

Process 4-1 Adiabatic Compression:

The cold body is removed and an insulating cap is brought in contact with the cylinder head. The piston moves to the TDC compressing the air adiabatically from temperature T_4 to T_1

W.K.T for adiabatic process, heat transfer Q = 0 and

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \longrightarrow (4)$$

To find air standard efficiency (η_{air}):

W.K.T efficiency $\eta_{air} = \frac{\text{Work done (WD)}}{\text{Heat supplied } (Q_H)}$

$$= \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}}$$

$$= \frac{Q_S - Q_r}{Q_S}$$

(or) $$\eta_{air} = 1 - \frac{Q_r}{Q_S} = 1 - \frac{m.RT_3 \ln\left(\frac{V_3}{V_4}\right)}{m.RT_1 \ln\left(\frac{V_2}{V_1}\right)} \longrightarrow (5)$$

Consider Eqn (2) $\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$

But, from t-s diagram, $T_2 = T_1$ and $T_3 = T_4$

$$\therefore \frac{T_1}{T_4} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \longrightarrow (6)$$

Comparing Eqn (4) and (6), we have $\left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$

$$\frac{V_1}{V_4} = \frac{V_2}{V_3} \text{ (or) } \frac{V_3}{V_4} = \frac{V_2}{V_1} \longrightarrow (7)$$

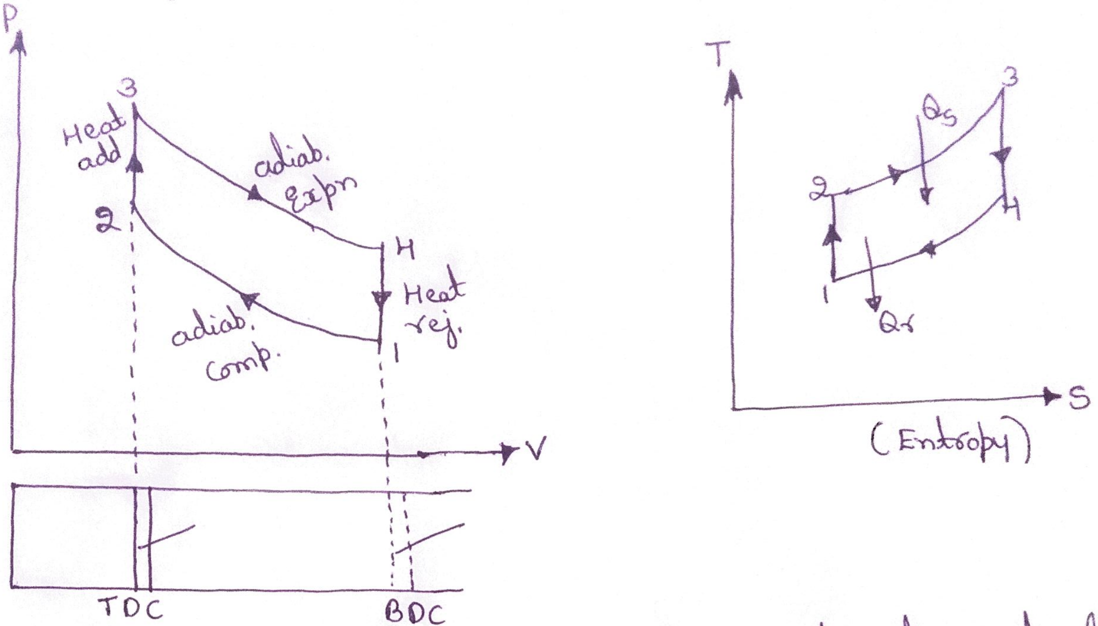
substituting Eqn (7) in (5), we get,

$$\eta_{air} = 1 - \frac{mRT_3 \ln\left(\frac{V_2}{V_1}\right)}{mRT_1 \ln\left(\frac{V_2}{V_1}\right)} = \boxed{1 - \frac{T_3}{T_1} = 1 - \frac{T_L}{T_H}}$$

$T_L = T_3 = T_4$ = Lowest temp. & $T_H = T_1 = T_2$ = Highest temp.

Otto Cycle:- Expression for Thermal Efficiency :-

Otto cycle Consists of four processes as shown on p-v and t-s diagrams.



Let the cylinder contain 'm' kg of air at its initial Condition represented by point 1 on p-v and t-s diagrams.

Process 1-2 Adiabatic Compression :

During this process, the insulating Cap is brought in Contact with the cylinder head. The piston moves from BDC to TDC Compressing the air adiabatically in the cylinder. The temperature of air rises from T_1 to T_2.

W.K.T for adiabatic process,

heat transfer $Q = 0$, and $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $\longrightarrow (1)$

Process 2-3 Constant Volume heat addition :-

Heat is supplied at Constant volume resulting in increase in pressure (P_3) and temperature (T_3).

Heat supplied $Q_s = mC_v(T_3 - T_2)$ ⟶ (2)

Process 3-4 Adiabatic Expansion :-

Air expands adiabatically and its temperature falls from T_3 to T_4. The piston moves towards BDC.

W.K.T for adiabatic process,

heat transfer $Q = 0$, and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ ⟶ (3)

Process 4-1 Constant Volume heat rejection:

Heat is rejected (transferred) to the cold body at constant volume.

Heat rejected $Q_r = mC_v(T_4 - T_1)$ ⟶ (4)

To find air standard efficiency (η_{air}):

W.K.T efficiency $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}}$

$$\eta_{air} = \frac{\text{Heat supplied} - \text{Heat rejected}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s}$$

i.e $\eta_{air} = 1 - \frac{Q_r}{Q_s} = 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$

$$\eta_{air} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \longrightarrow (5)$$

$$= 1 - \frac{\left(\frac{T_4}{T_1} - 1\right)T_1}{\left(\frac{T_3}{T_2} - 1\right)T_2} \longrightarrow (6)$$

From Eqn (1), We have $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ But $V_2 = V_3$ and $V_1 = V_4$

$\therefore \frac{T_1}{T_2} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$ (or) $\frac{T_2}{T_1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ ⟶ (7)

Comparing Eq^n (7) and (3), we have $\frac{T_3}{T_4} = \frac{T_2}{T_1}$

(or) $\frac{T_4}{T_1} = \frac{T_3}{T_2}$ $\longrightarrow$ (8)

Substituting Eq^n (8) in (6), we have

$$\eta_{air} = 1 - \frac{\left(\frac{T_3}{T_2} - 1\right) T_1}{\left(\frac{T_3}{T_2} - 1\right) T_2} = 1 - \frac{T_1}{T_2} \longrightarrow (9)$$

Defining Compression ratio $R_c = \frac{V_1}{V_2}$, we have from Eq^n(1)

$$\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1} \quad \text{(or)} \quad \frac{T_1}{T_2} = \frac{1}{(R_c)^{\gamma-1}} \longrightarrow (10)$$

Substituting Eq^n (10) in (9), we get

$$\boxed{\eta_{air} = 1 - \frac{1}{(R_c)^{\gamma-1}}}$$ for otto cycle.

Mean Effective Pressure for otto Cycle :-

Mean Effective pressure (MEP) is defined as the mean (or) average pressure acting on the piston during the power stroke of the working cycle.

$$MEP = P_m = \frac{\text{Work done / cycle}}{\text{Swept Volume}}$$

$$= \frac{Q_S - Q_r}{V_1 - V_2} \quad \text{[Refer p-v diagram]}$$

$$= \frac{m C_v (T_3 - T_2) - m C_v (T_4 - T_1)}{V_1 - V_2}$$

$$MEP = \frac{m C_v [(T_3 - T_2) - (T_4 - T_1)]}{V_1 - V_2} \longrightarrow (1)$$

Express temperatures T_2, T_3 and T_4 in terms of T_1 :

For adiabatic process 1-2, we have

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \text{ (or) } \frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1} \quad \left[\because R_c = \frac{V_1}{V_2}\right]$$

$$\therefore T_2 = T_1 \cdot (R_c)^{\gamma-1} \longrightarrow (2)$$

For constant volume process 2-3, we have $\frac{P}{T}$ = constant

i.e $\frac{P_2}{T_2} = \frac{P_3}{T_3}$ (or) $\frac{P_3}{P_2} = \frac{T_3}{T_2}$

$$\therefore T_3 = T_2 \cdot \left(\frac{P_3}{P_2}\right) = T_2 \cdot \alpha$$

Where α = Explosion ratio (or) pressure ratio $= \frac{P_3}{P_2}$

(or) $T_3 = T_1 (R_c)^{\gamma-1} \cdot \alpha$ from Eq^n (2) $\longrightarrow (3)$

For adiabatic process 3-4, we have, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$

But $V_4 = V_1$ & $V_3 = V_2$

$$\therefore \frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \text{ (or) } \frac{T_3}{T_4} = (R_c)^{\gamma-1}$$

$$T_4 = \frac{T_3}{(R_c)^{\gamma-1}} = \frac{T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha}{(R_c)^{\gamma-1}} \quad \text{from Eq}^n \text{ (3)}$$

$$\therefore T_4 = T_1 \cdot \alpha \longrightarrow (4)$$

Substituting Eq^n (2) & (3) and (4) in (1) we have

$$MEP = \frac{m C_v \left\{ \left[T_1 (R_c)^{\gamma-1} \cdot \alpha - T_1 (R_c)^{\gamma-1}\right] - (T_1 \alpha - T_1) \right\}}{(V_1 - V_2)}$$

$$= \frac{m C_v \cdot T_1 \left[(\alpha \cdot R_c^{\gamma-1} - R_c^{\gamma-1}) - (\alpha - 1) \right]}{V_1 - V_2} \longrightarrow (5)$$

under ideal conditions, at point (1), we have $P_1 V_1 = m R T_1$

(or) $V_1 = \frac{m R T_1}{P_1} \longrightarrow (6)$

W.K.T Compression ratio $R_c = \frac{V_1}{V_2}$

$$\therefore V_2 = \frac{V_1}{R_c} = \frac{mRT_1}{P_1 R_c} \text{ from (Eq}^n\text{ 6)} \longrightarrow (7)$$

Substituting Eq^n (6) & (7) in (5), we have

$$MEP = \frac{m.C_v.T_1\left[(\alpha R_c^{\gamma-1} - R_c^{\gamma-1}) - (\alpha-1)\right]}{\frac{mRT_1}{P_1} - \frac{mRT_1}{P_1 R_c}}$$

$$= \frac{m.C_v.T_1\left[R_c^{\gamma-1}(\alpha-1) - (\alpha-1)\right]}{\frac{mRT_1}{P_1}\left[1-\frac{1}{R_c}\right]}$$

$$= \frac{P_1 C_v (\alpha-1)\left[R_c^{\gamma-1} - 1\right]}{R\left[\frac{R_c-1}{R_c}\right]} \longrightarrow (8)$$

W.K.T Gas Constant $R = C_p - C_v$ (or) $\frac{R}{C_v} = \frac{C_p}{C_v} - 1$

$$\frac{R}{C_v} = \gamma - 1 \qquad \because \frac{C_p}{C_v} = \gamma$$

$$\therefore \frac{C_v}{R} = \frac{1}{\gamma-1} \longrightarrow (9)$$

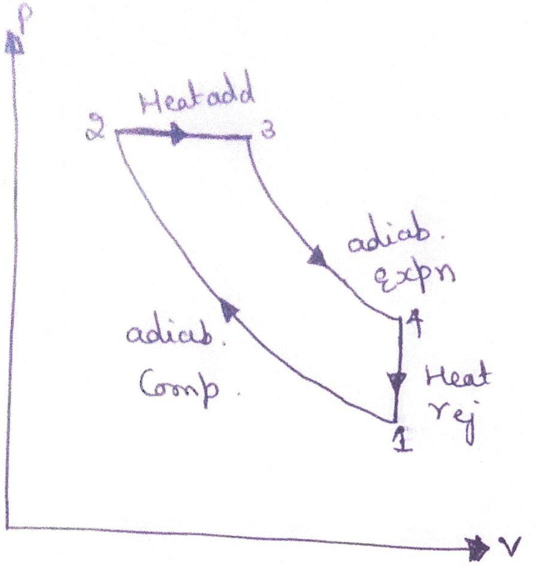
Substituting Eq^n (9) in (8), we have

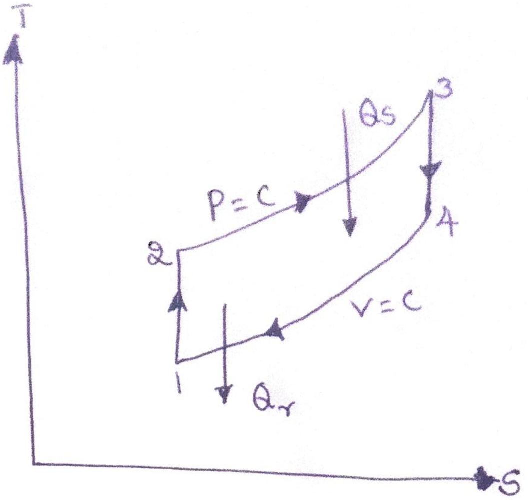
$$MEP = P_m = \frac{P_1 R_c (\alpha-1)(R_c^{\gamma-1}-1)}{(\gamma-1)(R_c-1)}$$

Diesel Cycle [Constant Pressure Cycle]

Diesel cycle consists of four processes as shown on P-V and T-S diagrams.

Let the cylinder contain m kg of air at its initial condition represented by point 1 on P-V and t-s diagrams





Process 1-2 Adiabatic Compression:

The piston moves from BDC to TDC compressing the air adiabatically in the cylinder. The temperature of air rises from T_1 to T_2.

W.K.T. for adiabatic process, heat transfer $Q=0$,

and $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \longrightarrow (1)$

Process 2-3 Constant pressure heat addition:

Heat is supplied at constant pressure resulting in an increase in the temperature from T_2 to T_3. At point 3, the supply of heat is stopped and this point is called as cut off.

∴ Heat supplied at constant pressure $= Q_S = mC_p\,(T_3 - T_2) \longrightarrow (2)$

Process 3-4 Adiabatic Expansion:

Air expands adiabatically and its temperature falls from T_3 to T_4. The piston moves towards BDC.

W.K.T for adiabatic process,

heat transfer $Q = 0$ and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ ⟶ (3)

Process 4-1 Constant Volume heat rejection:

Heat rejected to the cold body at Constant volume.

$\therefore$ Heat rejected $= Q_r = mC_v(T_4 - T_1)$ ⟶ (4)

To find Air standard Efficiency (η_{air}):

W.K.T $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$$\eta_{air} = 1 - \frac{m C_v (T_4 - T_1)}{m C_p (T_3 - T_2)}$$

But $\frac{C_p}{C_v} = \gamma$; $\therefore \eta_{air} = 1 - \frac{1}{\gamma}\frac{(T_4 - T_1)}{(T_3 - T_2)}$ ⟶ (5)

Express temperature T_2, T_3 and T_4 in terms of T_1

From Eqn (1), $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ $\because R_c = \frac{V_1}{V_2}$ = Compression Ratio.

$\therefore T_2 = T_1 (R_c)^{\gamma-1}$ ⟶ (6)

From Eqn (3), $\frac{T_3}{T_4} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$ $(\because V_4 = V_1)$

(or) $\frac{T_3}{T_4} = \left(\frac{V_1}{V_2}\cdot\frac{V_2}{V_3}\right)^{\gamma-1}$

Defining Cut-off ratio $\rho = \frac{V_3}{V_2}$, we have $\frac{T_3}{T_4} = R_c^{\gamma-1}\cdot\frac{1}{\rho^{\gamma-1}}$

(or) $T_4 = T_3 \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$ ⟶ (7)

Note that T_4 has to be Expressed in terms of T_1.

For Constant pressure process 2-3, $\frac{V}{T}$ = Constant i.e $\frac{V_2}{T_2} = \frac{V_3}{T_3}$ (or) $\frac{T_3}{T_2} = \frac{V_3}{V_2}$

$$T_3 = T_2 \left(\frac{V_3}{V_2}\right) = T_2 \rho$$

$$T_3 = T_1 (R_c)^{\gamma-1} \cdot \rho \quad \text{from Eqn (6)} \longrightarrow (8)$$

Substituting Eqn (8) in (7), we have $T_4 = T_1 (R_c)^{\gamma-1} \cdot \rho \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$

$$\therefore T_4 = T_1 \cdot \rho^{\gamma} \longrightarrow (9)$$

Substituting Eqn (6), (7), (8) and (9) in (5), we have

$$\eta_{air} = 1 - \frac{1}{\gamma} \frac{(T_1 \rho^{\gamma} - T_1)}{(T_1 R_c^{\gamma-1} \rho - T_1 R_c^{\gamma-1})}$$

$$\eta_{air} = 1 - \frac{1}{\gamma} \frac{T_1 (\rho^{\gamma} - 1)}{T_1 R_c^{\gamma-1} (\rho - 1)}$$

$$\boxed{\eta_{air} = 1 - \frac{1}{\gamma \cdot (R_c)^{\gamma-1}} \cdot \frac{(\rho^{\gamma} - 1)}{(\rho - 1)}}$$

Mean Effective Pressure for Disel cycle :-

Mean Effective pressure, $P_m = \frac{\text{Work done / Cycle}}{\text{Swept Volume}}$

$$= \frac{Q_s - Q_r}{V_1 - V_2} \quad \text{(Refer P-V diagram)}$$

$$\text{MEP}, P_m = \frac{m C_p (T_3 - T_2) - m C_v (T_4 - T_1)}{V_1 - V_2} \longrightarrow (1)$$

Express all temperature in terms of T_1 :

Reffering to Equations derived in section 2-6, we have heat transfer $Q = 0$ and $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$ $\longrightarrow$ (3)

Process 4-1 Constant Volume heat rejection:

Heat rejected to the cold body at constant volume.

$\therefore$ Heat rejected $= Q_r = mC_v(T_4 - T_1)$ $\longrightarrow$ (4)

To find Air standard Efficiency (η_{air}):

W.K.T $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$$\eta_{air} = 1 - \frac{m.C_v(T_4 - T_1)}{m.C_p(T_3 - T_2)}$$

But $\frac{C_p}{C_v} = \gamma$; $\therefore \eta_{air} = 1 - \frac{1(T_4 - T_1)}{\gamma(T_3 - T_2)}$ $\longrightarrow$ (5)

Express temperature T_2, T_3 and T_4 in terms of T_1

From Eqn (1), $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ $\quad \because R_c = \frac{V_1}{V_2}$ = Compression Ratio.

$\therefore T_2 = T_1(R_c)^{\gamma-1}$ $\longrightarrow$ (6)

From Eqn (3), $\frac{T_3}{T_4} = \left(\frac{V_1}{V_3}\right)^{\gamma-1}$ $\quad (\because V_4 = V_1)$

(or) $\frac{T_3}{T_4} = \left(\frac{V_1}{V_2} \cdot \frac{V_2}{V_3}\right)^{\gamma-1}$

Defining cut-off ratio $\rho = \frac{V_3}{V_2}$, we have $\frac{T_3}{T_4} = R_c^{\gamma-1} \cdot \frac{1}{\rho^{\gamma-1}}$

(or) $T_4 = T_3 \cdot \frac{\rho^{\gamma-1}}{(R_c)^{\gamma-1}}$ $\longrightarrow$ (7)

Note that T_4 has to be expressed in terms of T_1.

For constant pressure process 2-3, $\frac{V}{T}$ = Constant

From Eqn (6), $T_2 = T_1 (R_c)^{\gamma-1}$

From Eqn (8), $T_3 = T_1 R_c^{\gamma-1} \cdot \rho$

From Eqn (9), $T_4 = T_1 \cdot \rho^{\gamma}$

Substituting T_2, T_3 and T_4 in Eqn (1), we have

$$MEP = \frac{\left[m C_p \left(T_1 \cdot R_c^{\gamma-1} \cdot \rho - T_1 \cdot R_c^{\gamma-1} \right) - m C_v \left(T_1 \rho^{\gamma} - T_1 \right) \right]}{V_1 - V_2}$$

$$MEP = \frac{\left[m C_p \left(T_1 R_c^{\gamma-1} (\rho - 1) \right) - m C_v T_1 \left(\rho^{\gamma} - 1 \right) \right]}{V_1 - V_2} \longrightarrow (2)$$

Under ideal conditions, at point 1 on P-V diagram we have $P_1 V_1 = m R T_1$

$$\therefore V_1 = \frac{m R T_1}{P_1} \longrightarrow (3)$$

W.K.T Compression ratio, $R_c = \frac{V_1}{V_2}$

$$\therefore V_2 = \frac{V_1}{R_c} = \frac{m R T_1}{P_1 R_c} \quad \text{from Eqn (3)}$$

$$\therefore V_1 - V_2 = \frac{m R T_1}{P_1} - \frac{m R T_1}{P_1 R_c}$$

$$V_1 - V_2 = \frac{m R T_1}{P_1} \left(1 - \frac{1}{R_c} \right) \longrightarrow (4)$$

Substituting Eqn (4) in (2) we have,

$$MEP = \frac{m \cdot C_p \cdot T_1 R_c^{\gamma-1} (\rho - 1) - m C_v T_1 (\rho^{\gamma} - 1)}{\frac{m R T_1}{P_1} \left(1 - \frac{1}{R_c} \right)}$$

$$= \frac{P_1 R_c \left\{ C_p R_c^{\gamma-1} (\rho - 1) - C_v (\rho^{\gamma} - 1) \right\}}{R [R_c - 1]}$$

Divide both Numerator and Denominator by C_v

$$MEP = \frac{\frac{P_1 R_c}{C_v}\left\{C_p \cdot R_c^{\gamma-1}(\rho-1) - C_v(\rho^{\gamma}-1)\right\}}{\frac{R}{C_v}[R_c - 1]}$$

$$MEP = \frac{P_1 R_c\left\{\gamma R_c^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right\}}{\frac{R}{C_v}(R_c-1)}$$

$$\because \frac{C_p}{C_v} = \gamma \longrightarrow (5)$$

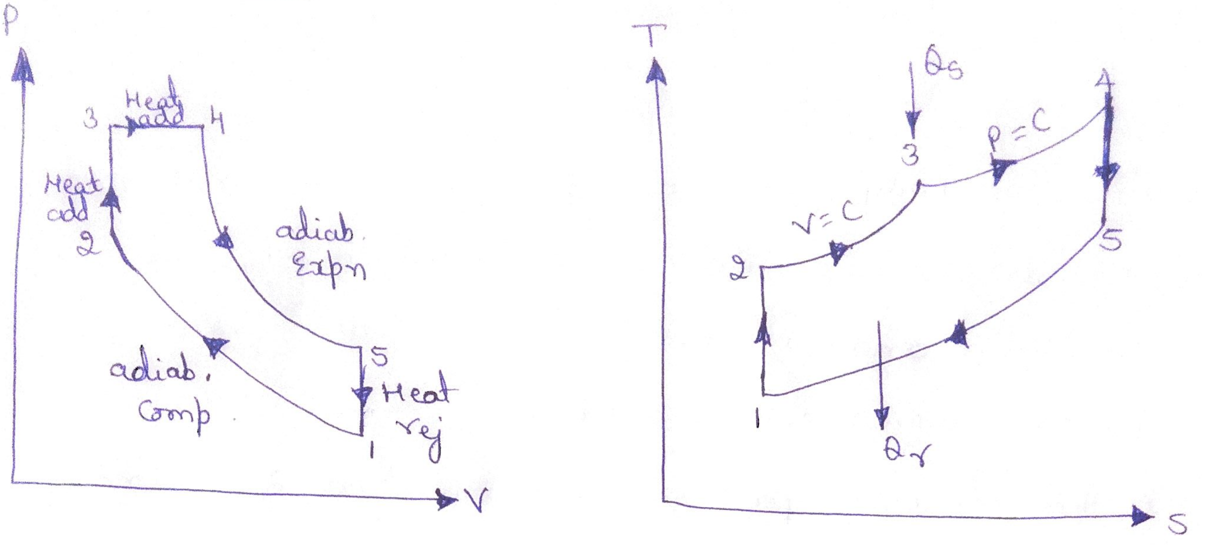
W.K.T $R = C_p - C_v$

$$\frac{R}{C_v} = \frac{C_p}{C_v} - 1 \quad \text{or} \quad \frac{R}{C_v} = \gamma - 1$$

$\therefore$ Eqn (5) becomes,

$$MEP = \frac{P_1 R_c\left\{\gamma R_c^{\gamma-1}(\rho-1) - (\rho^{\gamma}-1)\right\}}{(\gamma-1)(R_c-1)}$$

Dual Combustion Cycle (Semi-diesel cycle (or) Limited Pressure cycle):



Let the cylinder contain m kg of air at its initial condition represented by point 1 on p-v and t-s diagrams. The dual combustion cycle consists of the following processes.

Process 1-2 Adiabatic Compression:

The insulating cap is brought in contact with the cylinder head. The piston moves from BDC to TDC compressing the air adiabatically in the cylinder.

W.K.t for adiabatic process, heat transfer Q=0 and $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $\longrightarrow (1)$

Process 2-3 Constant Volume heat addition:

The insulating cap is removed and the hot body is brought in contact with the cylinder head. Heat is supplied at constant volume up to point 3.

$\therefore$ Heat supplied $(Q_s)_{2-3} = mC_v(T_3 - T_2)$ $\longrightarrow (2)$

Process 3-4 Constant pressure heat addition:

At point 3, heat is supplied at constant pressure up to condition 4 is reached.

$\therefore$ Heat supplied $(Q_s)_{3-4} = mC_p(T_4 - T_3)$ ⟶ (3)

Process 4-5 Adiabatic Expansion:

The hot body is removed and the insulating cap is brought in contact with the cylinder head. Air expands adiabatically and its temperature falls from T_4 and T_5. The piston moves towards BDC.

W.K.T for adiabatic process, heat transfer, $Q = 0$ and $\left(\frac{T_4}{T_5}\right) = \left(\frac{V_5}{V_4}\right)^{\gamma-1}$ ⟶ (4)

Process 5-1 Constant volume heat rejection:

The insulating cap is removed and the cold body is brought in contact with the cylinder head. Heat is rejected to the cold body at constant volume.

$\therefore$ Heat rejected $Q_r = mC_v(T_5 - T_1)$ ⟶ (5)

To find air standard efficiency (η_{air})

W.K.T $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_s - Q_r}{Q_s} = 1 - \frac{Q_r}{Q_s}$

$= 1 - \frac{Q_r}{(Q_s)_{2-3} + (Q_s)_{3-4}}$

i.e $\eta_{air} = 1 - \frac{mC_v(T_5 - T_1)}{mC_v(T_3 - T_2) + mC_p(T_4 - T_3)}$

$= 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$ ⟶ (6)

Express all temperatures in terms of T_1:

From Eqⁿ (1), we have $\frac{T_1}{T_2} = \left(\frac{1}{R_c}\right)^{\gamma-1}$ where $R_c = \frac{V_1}{V_2}$ = Compression ratio

$T_2 = T_1 (R_c)^{\gamma-1}$ ⟶ (7)

From Eqⁿ (4), we have $\frac{T_4}{T_5} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$ $\because V_5 = V_1$

$$\frac{T_4}{T_5} = \left(\frac{V_1}{V_2} \times \frac{V_2}{V_4}\right)^{\gamma-1}$$

(or) $\frac{T_4}{T_5} = \left(\frac{V_1}{V_2} \times \frac{V_3}{V_4}\right)^{\gamma-1}$ $\because V_2 = V_3$

$\frac{T_4}{T_5} = \left(R_c \times \frac{1}{\rho}\right)^{\gamma-1}$ $\because \frac{V_4}{V_3} = \rho$ cut-off ratio

$T_4 = T_5 \left(R_c^{\gamma-1} \cdot \rho^{\gamma-1}\right)$ ⟶ (8)

For constant volume process 2-3, we have $\frac{P}{T}$ = Constant

i.e $\frac{P_2}{T_2} = \frac{P_3}{T_3}$ (or) $T_3 = T_2 \left(\frac{P_3}{P_2}\right)$

$= T_2 (\alpha)$ where α = Explosion ratio $= \frac{P_3}{P_2}$

using Eqⁿ (7), we have, $T_3 = T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha$ ⟶ (9)

For constant pressure process 3-4, we have $\frac{V}{T}$ = Constant

i.e $\frac{V_4}{T_4} = \frac{V_3}{T_3}$ (or) $T_4 = T_3 \left(\frac{V_4}{V_3}\right) = T_3 (\rho)$

using Eqⁿ (9), we have $T_4 = T_1 \cdot (R_c)^{\gamma-1} \cdot \alpha \cdot \rho$ ⟶ (10)

(8)

Substituting Eqⁿ (10) in (8) we have $T_1 \cdot (R_c)^{\gamma-1} \times \alpha \times \rho = T_5 \times (R_c)^{\gamma-1} \times \rho^{1-\gamma}$

$$T_5 = \frac{T_1 \alpha \rho}{\rho^{1-\gamma}}$$

$$T_5 = T_1 \alpha \rho^{\gamma} \longrightarrow (11)$$

Substituting $T_2 \cdot T_3 \cdot T_4$ and T_5 in Eqⁿ (6), we have

$$\eta_{air} = 1 - \frac{C_v(T_1 \alpha \rho^{\gamma} - T_1)}{C_v(T_1 R_c^{\gamma-1} \alpha - T_1 R_c^{\gamma-1}) + C_p(T_1 R_c^{\gamma-1} \alpha \rho - T_1 R_c^{\gamma-1} \alpha)}$$

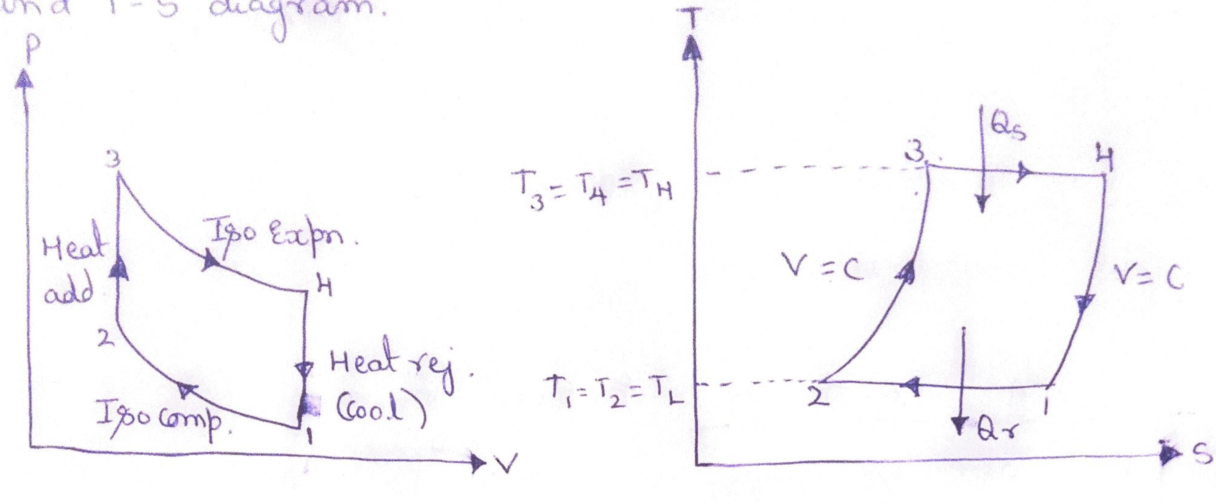
$$= 1 - \frac{T_1[\alpha \rho^{\gamma} - 1]}{T_1(R_c^{\gamma-1} \alpha - R_c^{\gamma-1}) + \frac{C_p}{C_v} \cdot T_1 (R_c^{\gamma-1} \cdot \alpha \cdot \rho - R_c^{\gamma-1} \cdot \alpha)}$$

$$\eta_{air} = 1 - \frac{\alpha \rho^{\gamma} - 1}{R_c^{\gamma-1}(\alpha - 1) + \gamma \cdot R_c^{\gamma-1} \cdot \alpha(\rho - 1)}$$

(or) $$\eta_{air} = 1 - \frac{1}{R_c^{\gamma-1}} \frac{\alpha \cdot \rho^{\gamma} - 1}{[(\alpha - 1) + \alpha \cdot \gamma(\rho - 1)]}$$

Stirling Cycle :-

Stirling cycle consists of four process as shown in P-V and T-S diagram.



Let us consider m kg of air at its initial conditions represented by point 1 on P-V and t-s diagrams. The various processes involved in stirling cycle are discussed below.

Process 1-2 Isothermal Compression :

During this process the cylinder head is kept in contact with the heat reservor at temperature T_1. The piston moves towards the TDC ; the air in the cylinder is compressed isothermally and heat is rejected from the working medium to the external sink.

$$\text{Heat rejected} = Q_r \text{ Work done} = P_1 V_1 \ln\left(\frac{V_1}{V_2}\right)$$

$$= mRT_1 \ln(R_c) \quad \Big\| \quad \because P_1 V_1 = mRT_1 \ \& \ \frac{V_1}{V_2} = R_C$$

Since $T_1 = T_L$ we can write, $Q_r = mRT_L \ln(R_c) \longrightarrow (1)$

Process 2-3 Constant volume heat addition :-

At condition 2, the air is made to enter the regenerator from its top. The air while passing through the regenerator matrix gets heated from T_2 to T_3 (T_L to T_H). During constant volume process, work done = 0.

But heat transfer $Q_{2-3} = mC_v(T_3 - T_2)$

Process 3-4 Isothermal Expansion :-

The hot air is now admitted to the engine cylinder from the bottom portion of the regenerator. Expansion of air takes place isothermally and work is produced. The piston moves towards BDC. During the process, the cylinder head is kept in contact with the heat source at T_H.

$$\text{Heat supplied } Q_s = \text{Workdone} = P_3V_3 \ln\left(\frac{V_4}{V_3}\right)$$

$$= mRT_3 \ln\left(\frac{V_1}{V_2}\right) \quad \| \because V_4 = V_1 \; \& \; V_3 = V_2$$

Since $T_3 = T_H$ we can write, $Q_s = mRT_H \ln(R_c)$.

Process 4-1 Constant volume heat rejection :-

At condition 4, the air is made to enter the regenerator from the bottom and gets cooled while passing through the regenerator matrix at constant volume.

The process is so controlled that ultimately the air comes to its initial condition 1 and the cycle is completed.

During constant volume process, work done = 0

∴ But, Heat transfer $Q_{4-1} = mC_v(T_4 - T_1)$

To find air standard efficiency (η_{air}):-

W.K.T $\eta_{air} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_S - Q_R}{Q_S} = 1 - \frac{Q_R}{Q_S}$

$$\eta_{air} = 1 - \frac{mRT_L \ln(R_C)}{mRT_H \ln(R_C)}$$

Thus for stirling cycles, $\eta_{air} = 1 - \frac{T_L}{T_H}$

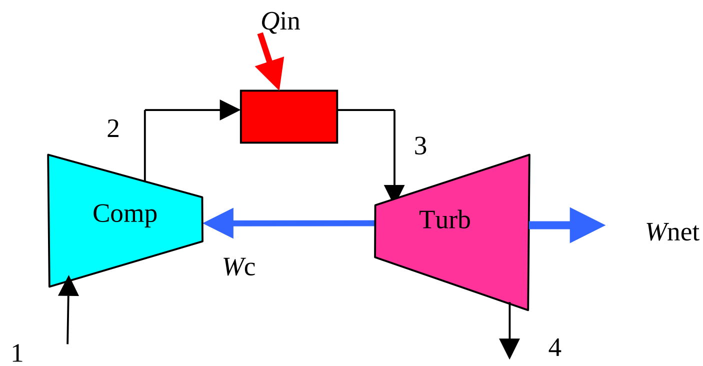
It is clear that the efficiency of stirling cycle is equal to that of Carnot cycle operating between the same temperature limits.

Brayton Cycle

The Brayton cycle is the air-standard ideal cycle approximation for the gas-turbine engine. This cycle differs from the Otto and Diesel cycles in that the processes making the cycle occur in open systems or control volumes. Therefore, an open system, steady-flow analysis is used to determine the heat transfer and work for the cycle.

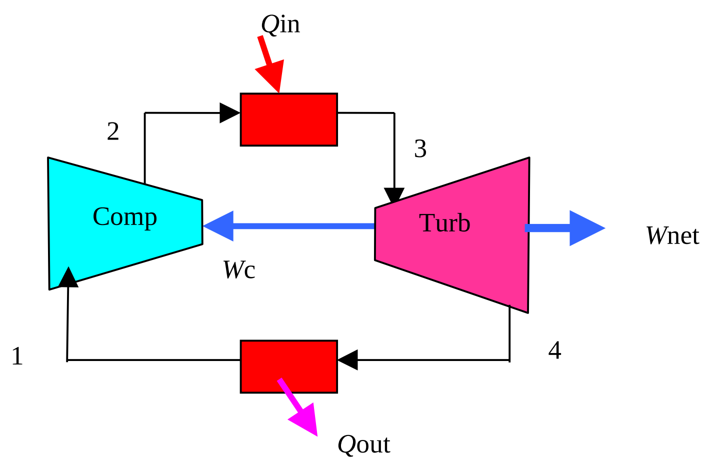
We assume the working fluid is air and the specific heats are constant and will consider the cold-air-standard cycle.

Brayton cycle



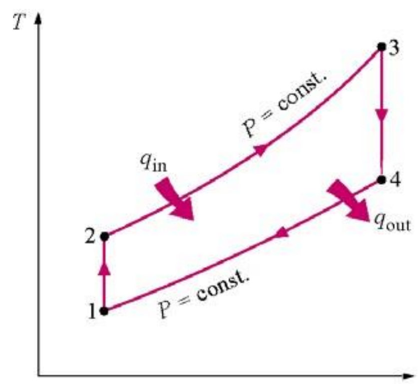
The closed cycle gas-turbine engine

Closed Brayton cycle

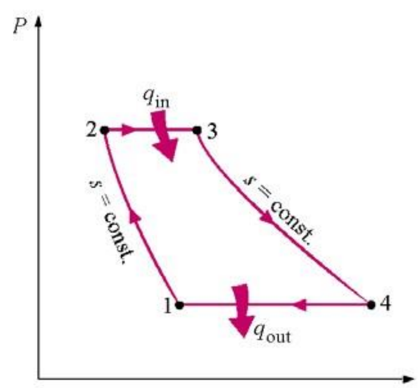


Process	Description
1-2	Isentropic compression (in a compressor)
2-3	Constant pressure heat addition
3-4	Isentropic expansion (in a turbine)
4-1	Constant pressure heat rejection

The *T-s* and *P-v* diagrams are



(a) *T-s* diagram



(b) *P-υ* diagram

Thermal efficiency of the Brayton cycle

$$\eta_{th,\ Brayton} = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Now to find Q_{in} and Q_{out}.

Apply the conservation of energy to process 2-3 for P = constant (no work), steady-flow, and neglect changes in kinetic and potential energies.

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

For constant specific heats, the heat added per unit mass flow is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$
$$\dot{Q}_{in} = \dot{m}C_p(T_3 - T_2)$$
$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = C_p(T_3 - T_2)$$

The conservation of energy for process 4-1 yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1)$$
$$\dot{Q}_{out} = \dot{m}C_p(T_4 - T_1)$$
$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = C_p(T_4 - T_1)$$

The thermal efficiency becomes

$$\eta_{th,\,Brayton} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p\,(T_4 - T_1)}{C_p\,(T_3 - T_2)}$$

$$\eta_{th,\,Brayton} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad and \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$, we see that

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

The Brayton cycle efficiency becomes

$$\eta_{th,\ Brayton} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot cycle efficiency?

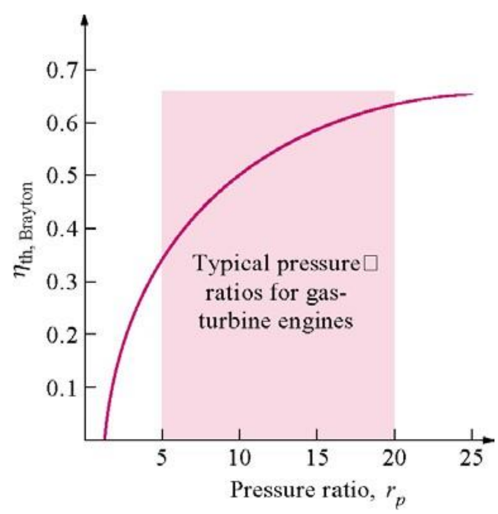
Since process 1-2 is isentropic,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

$$\frac{T_1}{T_2} = \frac{1}{r_p^{(k-1)/k}}$$

where the pressure ratio is $r_p = P_2/P_1$ and

$$\eta_{th,\ Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$



Extra Assignment

Evaluate the Brayton cycle efficiency by determining the net work directly from the turbine work and the compressor work. Compare your result with the above expression. Note that this approach does not require the closed cycle assumption.

Example 8-2

The ideal air-standard Brayton cycle operates with air entering the compressor at 95 kPa, 22°C. The pressure ratio r_p is 6:1 and the air leaves the heat addition process at 1100 K. Determine the compressor work and the turbine work per unit mass flow, the cycle efficiency, the back work ratio, and compare the compressor exit temperature to the turbine exit temperature. Assume constant properties.

Apply the conservation of energy for steady-flow and neglect changes in kinetic and potential energies to process 1-2 for the compressor. Note that the compressor is isentropic.

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_1 h_1 + \dot{W}_{comp} = \dot{m}_2 h_2$$

The conservation of mass gives

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

For constant specific heats, the compressor work per unit mass flow is

$$\dot{W}_{comp} = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{comp} = \dot{m}C_p(T_2 - T_1)$$

$$w_{comp} = \frac{\dot{W}_{comp}}{\dot{m}} = C_p(T_2 - T_1)$$

Since the compressor is isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k}$$

$$T_2 = T_1 r_p^{(k-1)/k}$$

$$= (22 + 273)K(6)^{(1.4-1)/1.4}$$

$$= 492.5\ K$$

$$w_{comp} = C_p(T_2 - T_1)$$

$$= 1.005\frac{kJ}{kg \cdot K}(492.5 - 295)\ K$$

$$= 198.15\frac{kJ}{kg}$$

The conservation of energy for the turbine, process 3-4, yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{W}_{turb} = \dot{m}(h_3 - h_4)$$

$$\dot{W}_{turb} = \dot{m}C_p(T_3 - T_4)$$

$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}} = C_p(T_3 - T_4)$$

Since process 3-4 is isentropic

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

Since $P_3 = P_2$ and $P_4 = P_1$, we see that

$$\frac{T_4}{T_3} = \left(\frac{1}{r_p}\right)^{(k-1)/k}$$

$$T_4 = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k}$$

$$= 1100K\left(\frac{1}{6}\right)^{(1.4-1)/1.4}$$

$$= 659.1K$$

$$w_{turb} = C_p(T_3 - T_4) = 1.005\frac{kJ}{kg \cdot K}(1100 - 659.1)K$$

$$= 442.5\frac{kJ}{kg}$$

We have already shown the heat supplied to the cycle per unit mass flow in process 2-3 is

$$\dot{m}_2 = \dot{m}_3 = \dot{m}$$

$$\dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

$$= C_p (T_3 - T_2) = 1.005 \frac{kJ}{kg \cdot K} (1100 - 492.5)\, K$$

$$= 609.6 \frac{kJ}{kg}$$

The net work done by the cycle is

$$w_{net} = w_{turb} - w_{comp}$$

$$= (442.5 - 198.15) \frac{kJ}{kg}$$

$$= 244.3 \frac{kJ}{kg}$$

The cycle efficiency becomes

$$\eta_{th,\ Brayton} = \frac{w_{net}}{q_{in}}$$

$$= \frac{244.3 \frac{kJ}{kg}}{609.6 \frac{kJ}{kg}} = 0.40 \quad or \quad 40\%$$

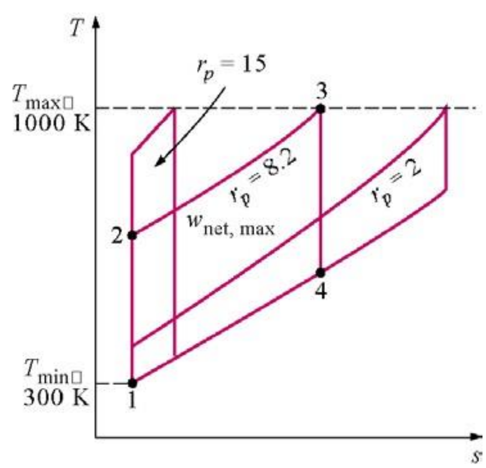
The back work ratio is defined as

$$BWR = \frac{w_{in}}{w_{out}} = \frac{w_{comp}}{w_{turb}}$$

$$= \frac{198.15 \frac{kJ}{kg}}{442.5 \frac{kJ}{kg}} = 0.448$$

Note that T_4 = 659.1 K > T_2 = 492.5 K, or the turbine outlet temperature is greater than the compressor exit temperature. Can this result be used to improve the cycle efficiency?

What happens to η_{th}, w_{in}/w_{out}, and w_{net} as the pressure ratio r_p is increased?



Let's take a closer look at the effect of the pressure ratio on the net work done.

$$w_{net} = w_{turb} - w_{comp}$$

$$= C_p (T_3 - T_4) - C_p (T_2 - T_1)$$

$$= C_p T_3 (1 - T_4 / T_3) - C_p T_1 (T_2 / T_1 - 1)$$

$$= C_p T_3 \left(1 - \frac{1}{r_p^{(k-1)/k}}\right) - C_p T_1 \left(r_p^{(k-1)/k} - 1\right)$$

Note that the net work is **zero** when

$$r_p = 1 \quad and \quad r_p = \left(\frac{T_3}{T_1}\right)^{(k-1)/k}$$

For fixed T_3 and T_1, the pressure ratio that makes the work a maximum is obtained from:

$$\frac{dw_{net}}{dr_p} = 0$$

This is easier to do if we let $X = r_p^{(k-1)/k}$

$$w_{net} = C_p T_3 \left(1 - \frac{1}{X}\right) - C_p T_1 (X - 1)$$

$$\frac{dw_{net}}{dX} = C_p T_3 [0 - (-1) X^{-2}] - C_p T_1 [1 - 0] = 0$$

Solving for X

$$X^2 = \frac{T_3}{T_1} = \left(r_p\right)^{2(k-1)/k}$$

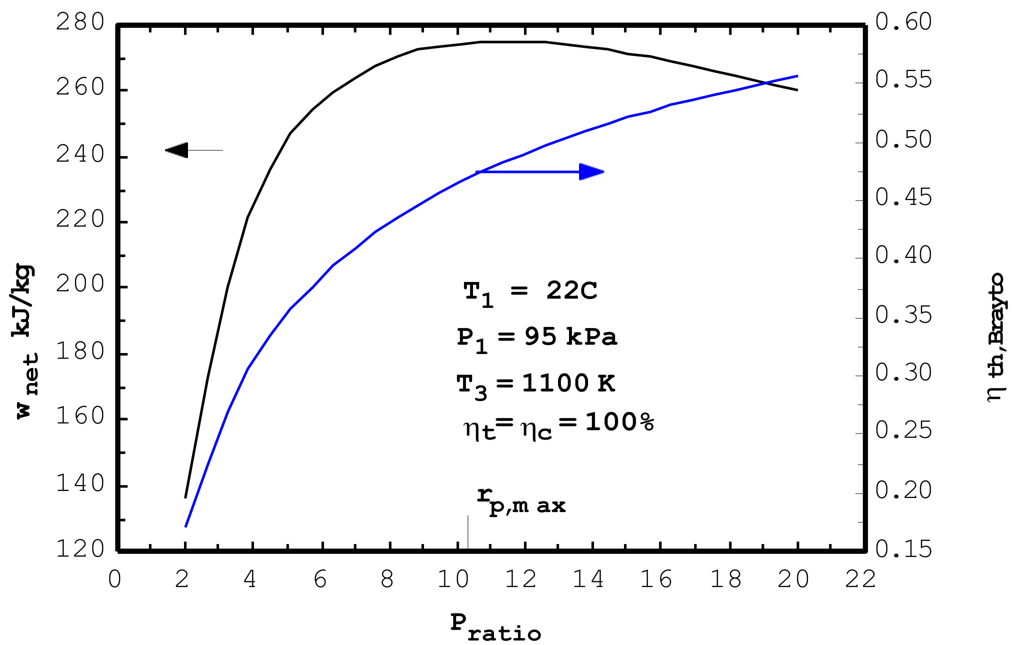
Then, the r_p that makes the work a maximum for the constant property case and fixed T_3 and T_1 is

$$r_{p,\text{ max work}} = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

For the ideal Brayton cycle, show that the following results are true.

- When $r_p = r_{p,\text{ max work}}$, $T_4 = T_2$
- When $r_p < r_{p,\text{ max work}}$, $T_4 > T_2$
- When $r_p > r_{p,\text{ max work}}$, $T_4 < T_2$

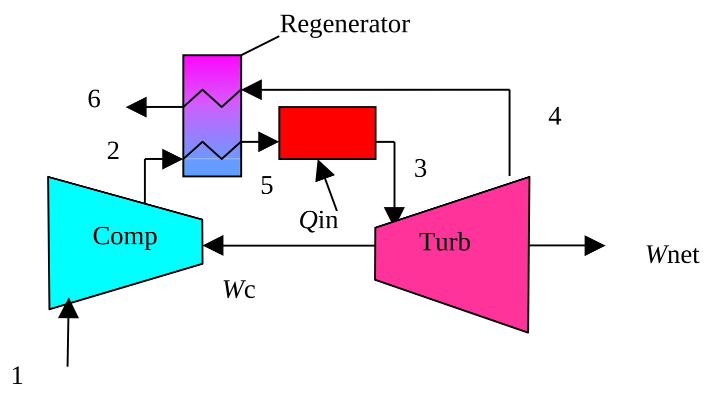
The following is a plot of net work per unit mass and the efficiency for the above example as a function of the pressure ratio.

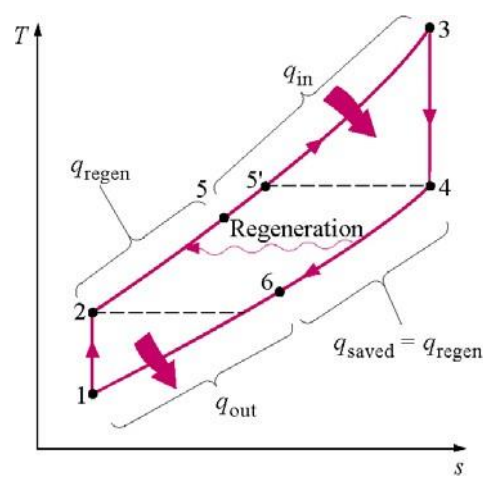


Regenerative Brayton Cycle

For the Brayton cycle, the turbine exhaust temperature is greater than the compressor exit temperature. Therefore, a heat exchanger can be placed between the hot gases leaving the turbine and the cooler gases leaving the compressor. This heat exchanger is called a regenerator or recuperator. The sketch of the regenerative Brayton cycle is shown below.

Regenerative Brayton cycle





We define the regenerator effectiveness ε_{regen} as the ratio of the heat transferred to the compressor gases in the regenerator to the maximum possible heat transfer to the compressor gases.

$$q_{regen,\,act} = h_5 - h_2$$

$$q_{regen,\,\max} = h_{5'} - h_2 = h_4 - h_2$$

$$\varepsilon_{regen} = \frac{q_{regen,\,act}}{q_{regen,\,\max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

For ideal gases using the cold-air-standard assumption with constant specific heats, the regenerator effectiveness becomes

$$\varepsilon_{regen} \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Using the closed cycle analysis and treating the heat addition and heat rejection as steady-flow processes, the regenerative cycle thermal efficiency is

$$\eta_{th,\,regen} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{h_6 - h_1}{h_3 - h_5}$$

Notice that the heat transfer occurring within the regenerator is **not** included in the efficiency calculation because this energy is not a heat transfer across the cycle boundary.

Assuming an ideal regenerator $\varepsilon_{regen} = 1$ and constant specific heats, the thermal efficiency becomes (take the time to show this on your own)

$$\eta_{th,\,regen} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

When does the efficiency of the air-standard Brayton cycle equal the efficiency of the air-standard regenerative Brayton cycle? If we set $\eta_{\text{th, Brayton}} = \eta_{\text{th, regen}}$ then

$$\eta_{th,\,Brayton} = \eta_{th,\,regen}$$

$$1 - \frac{1}{(r_p)^{(k-1)/k}} = 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

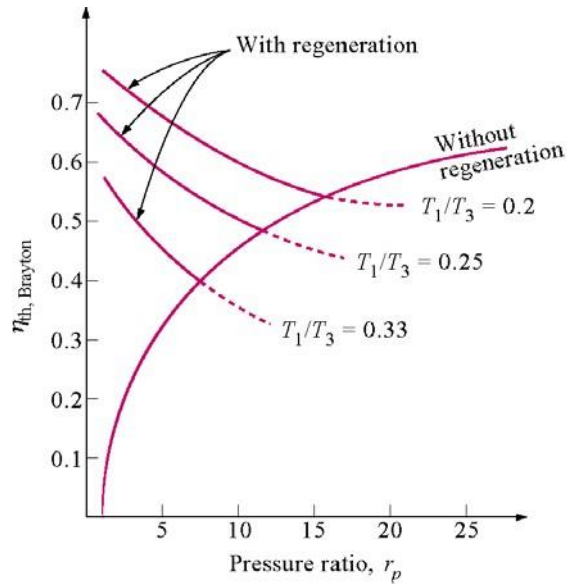
Recall that this is the pressure ratio that maximizes the net work for the simple Brayton cycle and makes $T_4 = T_2$. What happens if the regenerative Brayton cycle operates at a pressure ratio larger than this value?

For fixed T_3 and T_1, pressure ratios greater than this value cause T_4 to be less than T_2, and the regenerator is not effective.

What happens to the net work when a regenerator is added?

What happens to the heat supplied when a regenerator is added?

The following shows a plot of the regenerative Brayton cycle efficiency as a function of the pressure ratio and minimum to maximum temperature ratio, T_1/T_3.



Example 8-3: Regenerative Brayton Cycle

Air enters the compressor of a regenerative gas-turbine engine at 100 kPa and 300 K and is compressed to 800 kPa. The regenerator has an effectiveness of 65 percent, and the air enters the turbine at 1200 K. For a compressor efficiency of 75 percent and a turbine efficiency of 86 percent, determine

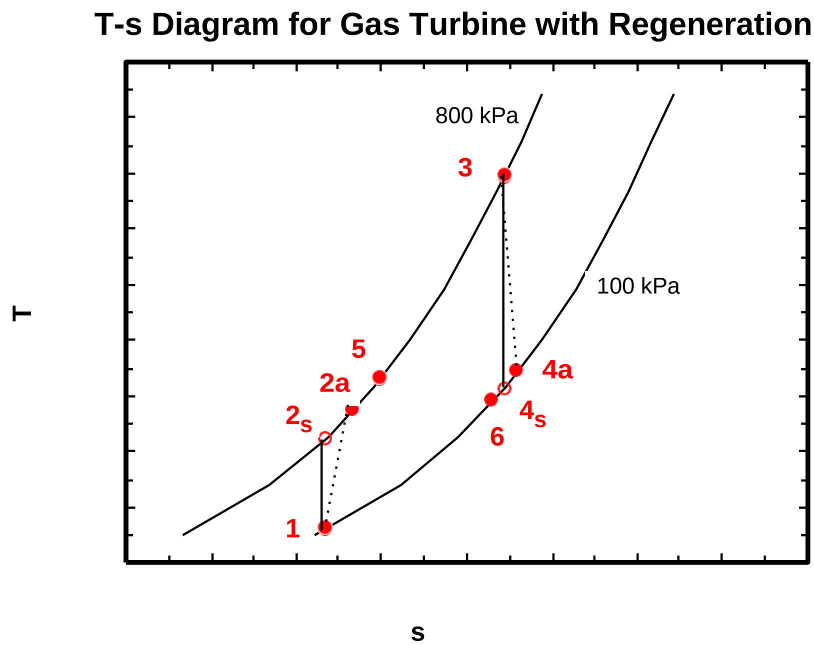
(a) The heat transfer in the regenerator.
(b) The back work ratio.
(c) The cycle thermal efficiency.

Compare the results for the above cycle with the ones listed below that have the same common data as required.

(a) The actual cycle with no regeneration, ε = 0.
(b) The actual cycle with ideal regeneration, ε = 1.0.
(b) The ideal cycle with regeneration, ε = 0.65.
(d) The ideal cycle with no regeneration, ε = 0.
(e) The ideal cycle with ideal regeneration, ε = 1.0.

We assume air is an ideal gas with constant specific heats, that is, we use the cold-air-standard assumption.

The cycle schematic is the same as above and the *T-s* diagram showing the effects of compressor and turbine efficiencies is below.



Summary of Results

Cycle type	Actual	Actual	Actual	Ideal	Ideal	Ideal
ε_{regen}	0.00	0.65	1.00	0.00	0.65	1.00
η_{comp}	0.75	0.75	0.75	1.00	1.00	1.00
η_{turb}	0.86	0.86	0.86	1.00	1.00	1.00
q_{in} kJ/kg	578.3	504.4	464.6	659.9	582.2	540.2
w_{comp} kJ/kg	326.2	326.2	326.2	244.6	244.6	244.6
w_{turb} kJ/kg	464.6	464.6	464.6	540.2	540.2	540.2
w_{comp}/w_{turb}	0.70	0.70	0.70	0.453	0.453	0.453
η_{th}	24.0%	27.5%	29.8%	44.8%	50.8%	54.7%

Compressor analysis

The isentropic temperature at compressor exit is

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 300K\left(\frac{800kPa}{100kPa}\right)^{(1.4-1)/1.4} = 543.4K$$

To find the actual temperature at compressor exit, T_{2a}, we apply the compressor efficiency

$$\eta_{comp} = \frac{w_{isen,comp}}{w_{act,comp}} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \cong \frac{T_{2s} - T_1}{T_{2a} - T_1}$$

$$T_{2a} = T_1 + \frac{1}{\eta_{comp}}(T_{2s} - T_1)$$

$$= 300K + \frac{1}{0.75}(543.4 - 300)K$$

$$= 624.6K$$

Since the compressor is adiabatic and has steady-flow

$$w_{comp} = h_{2a} - h_1 = C_p(T_{2a} - T_1)$$

$$= 1.005\frac{kJ}{kg \cdot K}(624.6 - 300)K = 326.2\frac{kJ}{kg}$$

Turbine analysis

The conservation of energy for the turbine, process 3-4, yields for constant specific heats (let's take a minute for you to get the following result)

$$\dot{W}_{turb} = \dot{m}(h_3 - h_{4a})$$

$$\dot{W}_{turb} = \dot{m}C_p(T_3 - T_{4a})$$

$$w_{turb} = \frac{\dot{W}_{turb}}{\dot{m}} = C_p(T_3 - T_{4a})$$

Since $P_3 = P_2$ and $P_4 = P_1$, we can find the isentropic temperature at the turbine exit.

$$\frac{T_{4s}}{T_3} = \left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k}$$

$$= 1200K\left(\frac{100kPa}{800kPa}\right)^{(1.4-1)/1.4} = 662.5K$$

To find the actual temperature at turbine exit, T_{4a}, we apply the turbine efficiency.

$$\eta_{turb} = \frac{w_{act,turb}}{w_{isen,turb}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}} \cong \frac{T_3 - T_{4a}}{T_3 - T_{4s}}$$

$$T_{4a} = T_3 - \eta_{turb}(T_3 - T_{4s})$$
$$= 1200K - 0.86(1200 - 662.5)\,K$$
$$= 737.7K > T_{2a}$$

The turbine work becomes

$$w_{turb} = h_3 - h_{4a} = C_p(T_3 - T_{4a})$$
$$= 1.005\frac{kJ}{kg \cdot K}(1200 - 737.7)K$$
$$= 464.6\frac{kJ}{kg}$$

The back work ratio is defined as

$$BWR = \frac{w_{in}}{w_{out}} = \frac{w_{comp}}{w_{turb}}$$
$$= \frac{326.2\frac{kJ}{kg}}{464.6\frac{kJ}{kg}} = 0.70$$

Regenerator analysis

To find T_5, we apply the regenerator effectiveness.

$$\varepsilon_{regen} \cong \frac{T_5 - T_{2a}}{T_{4a} - T_{2a}}$$

$$T_5 = T_{2a} + \varepsilon_{regen}(T_{4a} - T_{2a})$$

$$= 624.6K + 0.65(737.7 - 624.6)K$$

$$= 698.1K$$

To find the heat transferred from the turbine exhaust gas to the compressor exit gas, apply the steady-flow conservation of energy to the compressor gas side of the regenerator.

$$\dot{m}_{2a} h_{2a} + \dot{Q}_{regen} = \dot{m}_5 h_5$$

$$\dot{m}_{2a} = \dot{m}_5 = \dot{m}$$

$$q_{regen} = \frac{\dot{Q}_{regen}}{\dot{m}} = h_5 - h_{2a}$$

$$= C_p(T_5 - T_{2a})$$

$$= 1.005 \frac{kJ}{kg \cdot K}(698.1 - 624.6)K$$

$$= 73.9 \frac{kJ}{kg}$$

Using q_{regen}, we can determine the turbine exhaust gas temperature at the regenerator exit.

$$\dot{m}_{4a} h_{4a} = \dot{Q}_{regen} + \dot{m}_6 h_6$$

$$\dot{m}_{4a} = \dot{m}_6 = \dot{m}$$

$$q_{regen} = \frac{\dot{Q}_{regen}}{\dot{m}} = h_{4a} - h_6 = C_p (T_{4a} - T_6)$$

$$T_6 = T_{4a} - \frac{q_{regen}}{C_p} = 737.7K - \frac{73.9 \frac{kJ}{kg}}{1.005 \frac{kJ}{kg \cdot K}}$$

$$= 664.2K$$

Heat supplied to cycle

Apply the steady-flow conservation of energy to the heat exchanger for process 5-3. We obtain a result similar to that for the simple Brayton cycle.

$$q_{in} = h_3 - h_5 = C_p (T_3 - T_5)$$

$$= 1.005 \frac{kJ}{kg \cdot K} (1200 - 698.1) K$$

$$= 504.4 \frac{kJ}{kg}$$

Cycle thermal efficiency

The net work done by the cycle is

$$w_{net} = w_{turb} - w_{comp}$$

$$= (464.6 - 326.2) \frac{kJ}{kg} = 138.4 \frac{kJ}{kg}$$

The cycle efficiency becomes

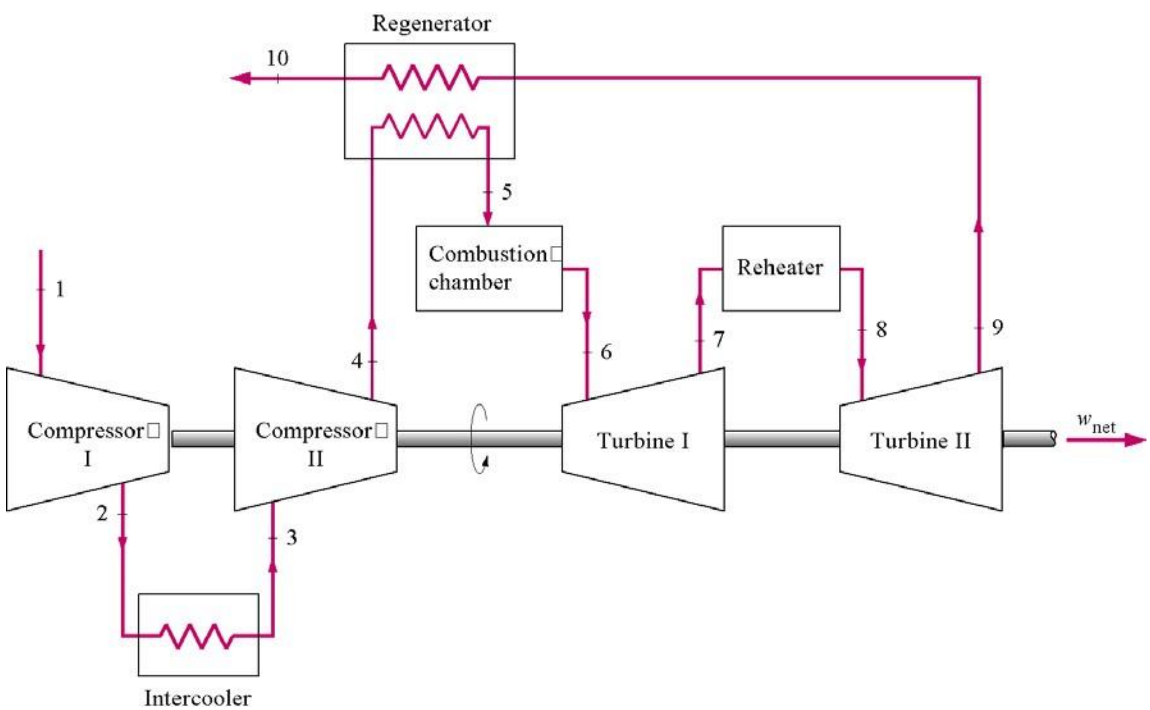
$$\eta_{th,\ Brayton} = \frac{w_{net}}{q_{in}}$$

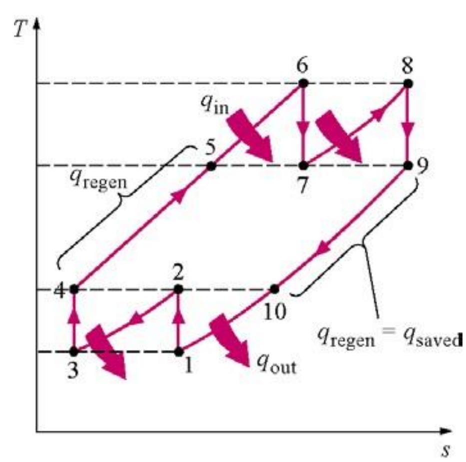
$$= \frac{138.4\ \frac{kJ}{kg}}{504.4\ \frac{kJ}{kg}} = 0.274 \quad or \quad 27.4\%$$

You are encouraged to complete the calculations for the other values found in the summary table.

Other Ways to Improve Brayton Cycle Performance

Intercooling and reheating are two important ways to improve the performance of the Brayton cycle with regeneration.





Intercooling

When using multistage compression, cooling the working fluid between the stages will reduce the amount of compressor work required. The compressor work is reduced because cooling the working fluid reduces the average specific volume of the fluid and thus reduces the amount of work on the fluid to achieve the given pressure rise.

To determine the intermediate pressure at which intercooling should take place to minimize the compressor work, we follow the approach shown in Chapter 6.

For the adiabatic, steady-flow compression process, the work input to the compressor per unit mass is

$$w_{comp} = \int_1^4 v\,dP = \int_1^2 v\,dP + \cancelto{0}{\int_2^3 v\,dP} + \int_3^4 v\,dP$$

For the isentropic compression process

$$w_{comp} = \frac{k}{k\text{-}1}(P_2 v_2 - P_1 v_1) + \frac{k}{k\text{-}1}(P_4 v_4 - P_3 v_3)$$

$$= \frac{k}{k\text{-}1} R(T_2 - T_1) + \frac{kR}{k\text{-}1}(T_4 - T_3)$$

$$= \frac{k}{k\text{-}1} R\left[T_1(T_2 / T_1 - 1) + T_3(T_4 / T_3 - 1)\right]$$

$$= \frac{k}{k\text{-}1} R\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$$

Notice that the fraction $kR/(k\text{-}1) = C_p$.

$$w_{comp} = C_p\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$$

Can you obtain this relation another way? Hint: apply the first law to processes 1-4.

For two-stage compression, let’s assume that intercooling takes place at constant pressure and the gases can be cooled to the inlet temperature for the compressor, such that $P_3 = P_2$ and $T_3 = T_1$.

The total work supplied to the compressor becomes

$$w_{comp} = C_p T_1 \left[\left(\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right) + \left(\left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 1 \right) \right]$$

$$= C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} + \left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 2 \right]$$

To find the unknown pressure P_2 that gives the minimum work input for fixed compressor inlet conditions T_1, P_1, and exit pressure P_4, we set

$$\frac{dw_{comp}(P_2)}{dP_2} = 0$$

This yields

$$P_2 = \sqrt{P_1 P_4}$$

or, the pressure ratios across the two compressors are equal.

$$\frac{P_2}{P_1} = \frac{P_4}{P_2} = \frac{P_4}{P_3}$$

Intercooling is almost always used with regeneration. During intercooling the compressor exit temperature is reduced; therefore, more heat must be supplied in the heat addition process. Regeneration can make up part of the required heat transfer.

To supply only compressed air, using intercooling requires less work input. The next time you go to a home supply store where air compressors are sold, check the larger air compressors to see if intercooling is used. For the larger air compressors, the compressors are made of two piston-cylinder chambers. The intercooling heat exchanger may be only a pipe with a few attached fins that connects the large piston-cylinder chamber with the smaller piston-cylinder chamber.

Reheating

When using multistage expansion through two or more turbines, reheating between stages will increase the net work done (it also increases the required heat input).The regenerative Brayton cycle with reheating is shown above.

The optimum intermediate pressure for reheating is the one that maximizes the turbine work. Following the development given above for intercooling and assuming reheating to the high-pressure turbine inlet temperature in a constant pressure steady-flow process, we can show the optimum reheat pressure to be

$$P_7 = \sqrt{P_6 P_9}$$

or the pressure ratios across the two turbines are equal.