ATME College of Engineering

CIVIL ENGINEERING

Module -1

Fluid and its properties

Fluid

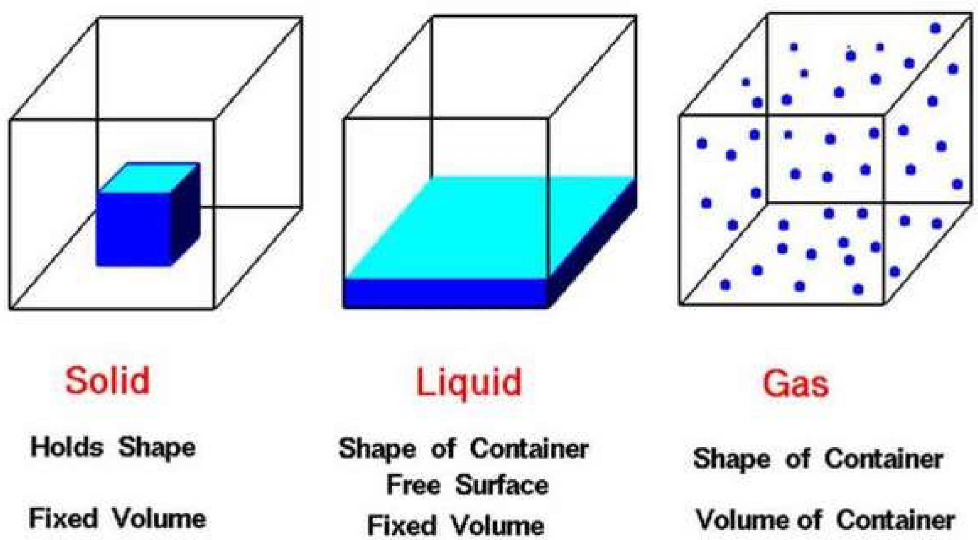
A fluid is a substance that continually deforms (flows) under an applied shear stress, or external force.

Fluids are a phase of matter and include liquids, gases and plasmas.

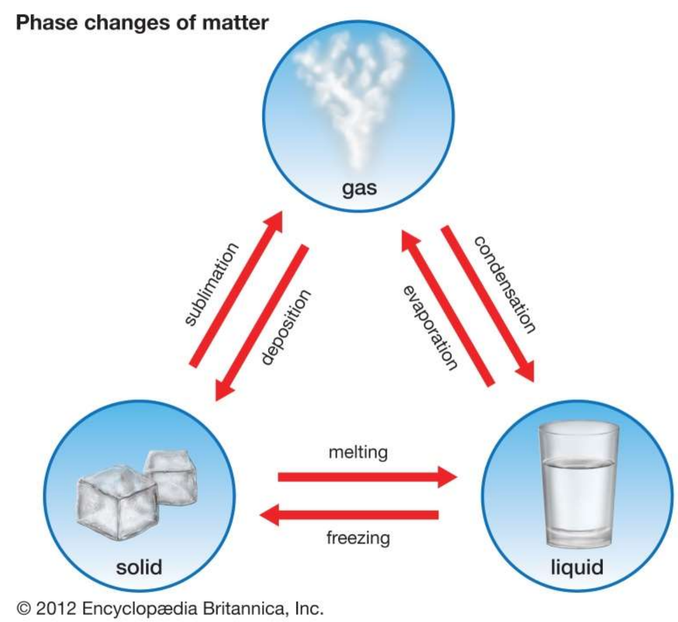
They are substances with zero shear modulus, or, in simpler terms, substances which cannot resist any shear force applied to them.

Fluid and Solid

- A *fluid is a substance that deforms continuously under the* application of a shear (tangential) stress no matter how small the shear stress may be.
- A solid deforms when a shear stress is applied, but its deformation does not continue to increase with time.



Source: www.grc.nasa.gov/



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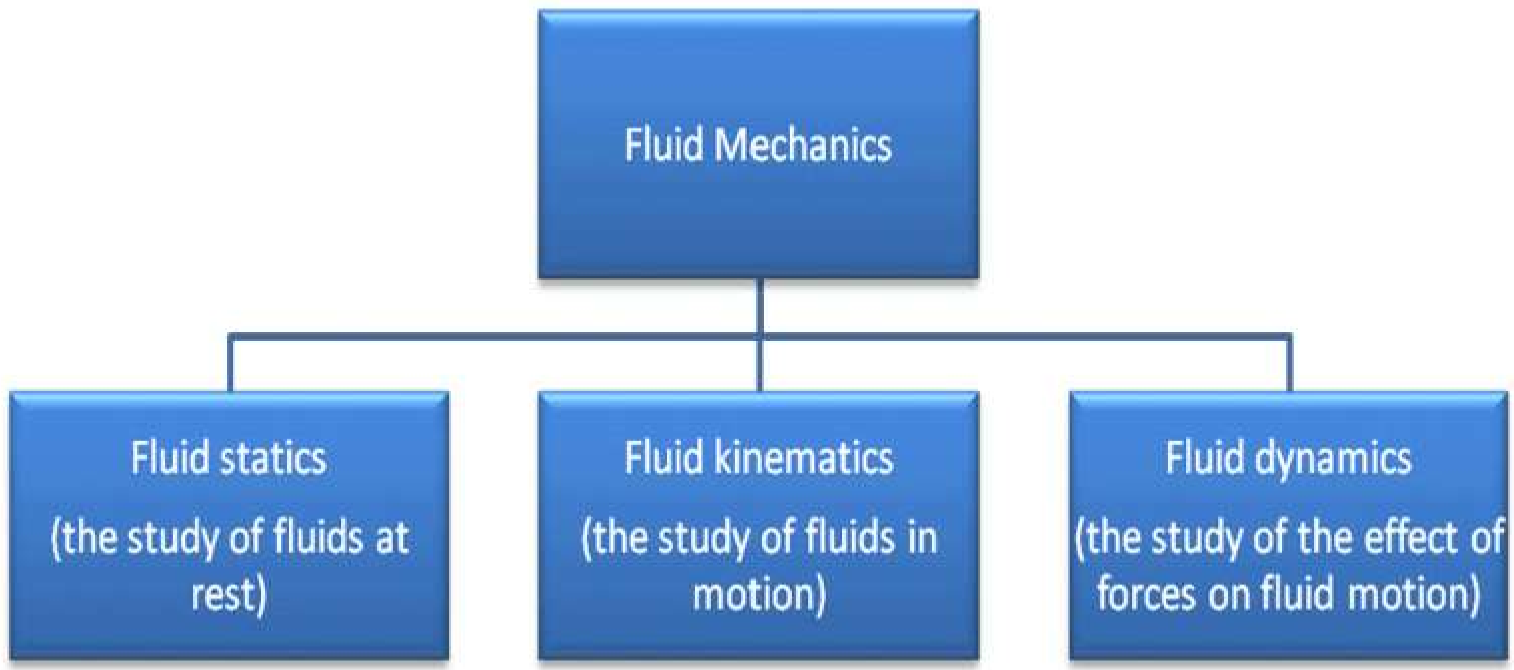
Fluid mechanics

- Fluid mechanics is the branch of physics concerned with the mechanics of fluids (liquids, gases, and plasmas) and the forces on them.



Source: http://ecoursesonline.iasri.res.in/mod/page/view.php?id=1064

Systems of Units:

Standard system: System International Units

Primary quantities	
Quantities	Unit
Mass	kg
Length	M
Time	S
Temperature in kelvin	K

Derived quantities	
Quantities	Unit
Force in Newton (1 N = 1 kg.m/s^2)	N
Pressure in Pascal (1 Pa = 1 N/m^2)	N/m^2
Work, energy in Joule (1 J = 1N.m)	J
Power in Watt (1 W = 1 J/s)	W

Properties of fluids

- Mass density or Specific mass or Density (ρ)
- Weight density or Specific weight(γ)
- Relative density or Specific gravity(S)
- Specific volume (V)
- Viscosity(μ)
- Surface Tension
- Capillarity

Mass density or Specific mass or Density (ρ)

Mass density or specific mass is defined as the mass per unit volume of the fluid. It is represented by Greek letter ρ.

$$\rho = \frac{Mass}{Volume}$$

With the increase in temperature volume of fluid increases and hence mass density decreases in case of fluids as the pressure increases volume decreases and hence mass density increases.

Weight density or Specific weight(γ)

Weight density or Specific weight of a fluid is defined as the ratio of weight per unit volume of the fluid. It is represented by Greek letter γ

$$\gamma = \frac{Weight}{Volume}$$

With increase in temperature volume increases and hence specific weight decreases. With increases in pressure volume decreases and hence specific weight increases.

Relative density or Specific gravity(S)

Defined as the ratio of density of the fluid to the density of a standard fluid.

$$S = \frac{Density\ of\ fluid}{density\ of\ standard\ fluid}$$

Unit: It is a dimensionless quantity hence no unit

Specific volume (∀)

It is defined as the volume of fluid per unit mass of the same fluid.

$$\forall = \frac{Volume}{Mass}$$

Unit: m^3 /kg

As the temperature increases volume increases and hence specific volume increases.
As the pressure increases volume decreases and hence specific volume decreases.

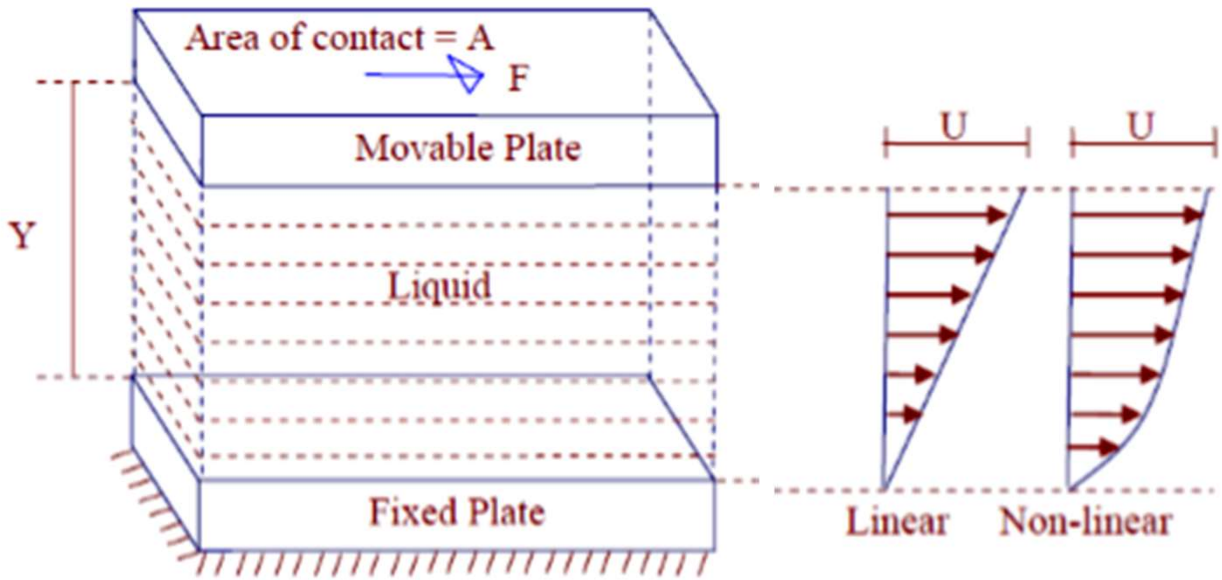
Viscosity

- Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow.
- Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.
- In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

Newton's law of viscosity

- **Statement:** The shear stress between adjacent fluid layers is proportional to the velocity gradients between the two layers.
- The ratio of shear stress to shear rate is a constant, for a given temperature and pressure, and is defined as the viscosity or coefficient of viscosity.

Newton's law of viscosity.....



Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart, 'A' is the contact area (Wetted area) of the movable plate, 'F' is the force required to move the plate with a velocity 'V'.

From experiments it has been observed that

$$F \propto A, F \propto \frac{1}{y}, F \propto V$$

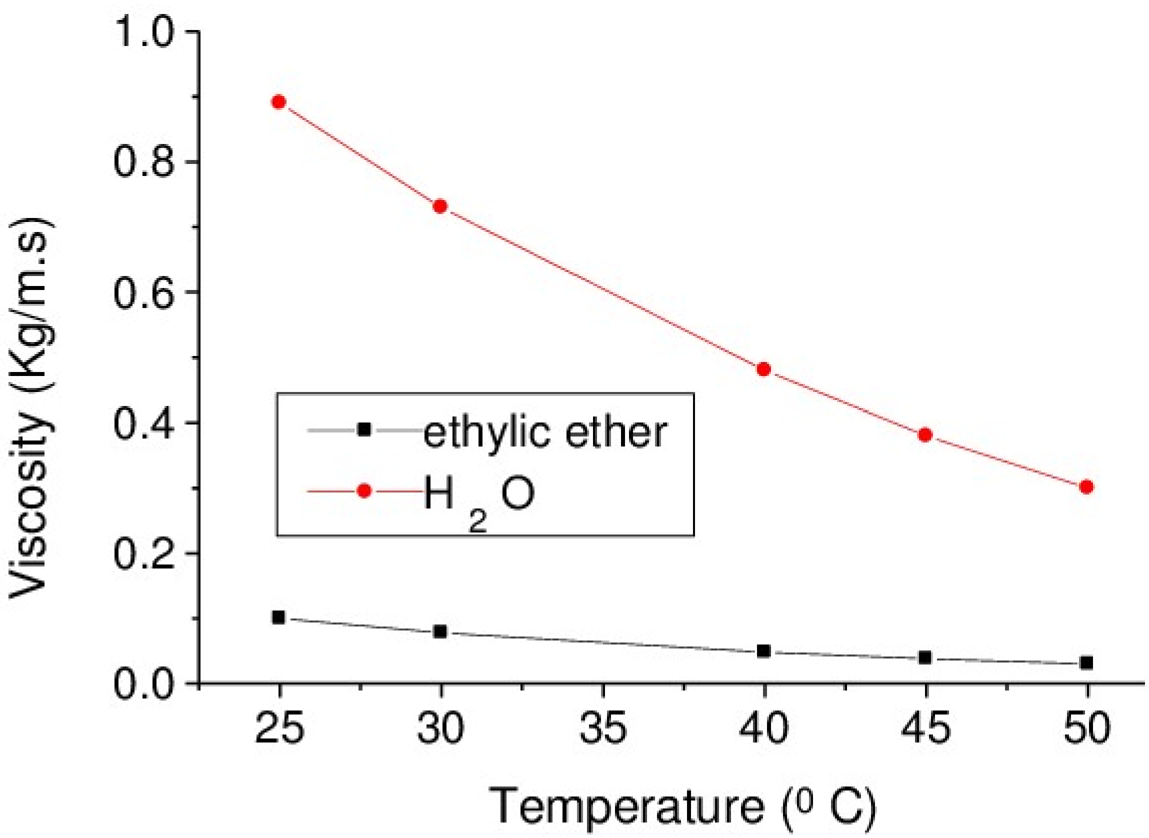
$$F \propto \frac{A \times V}{y}$$

$$F = \mu \frac{A \times V}{y}$$

$$\frac{F}{A} = \mu \frac{V}{y}$$

$$\tau = \mu \frac{dV}{dy}$$

Effect of temperature and pressure on viscosity



Source: Conference paper titled "Microfluidic dynamic system for biological fluids viscosity measurements"

Effect of temperature on viscosity of liquids: Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.

Effect of temperature on viscosity of gases: Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

Kinematics Viscosity(ν)

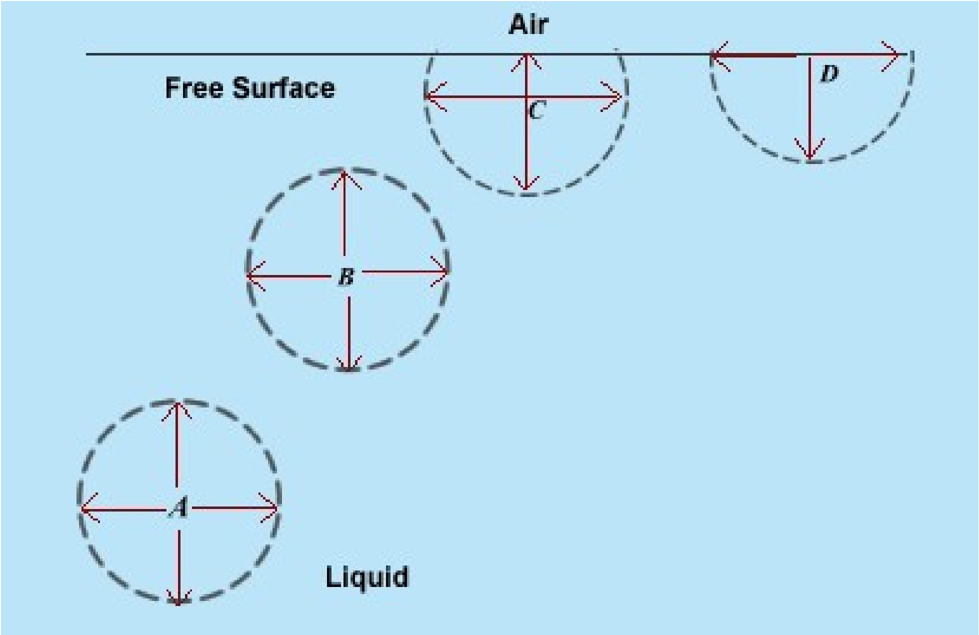
- It is the ratio of dynamic viscosity of the liquid to its mass density.

$$\nu = \frac{\mu}{\rho}$$

The dimensional analysis of the above ration yields only magnitudes of length and time and by definition kinematics involves normally the dimensions of length and time. Hence the name 'kinematic viscosity'

Surface Tension of Liquids

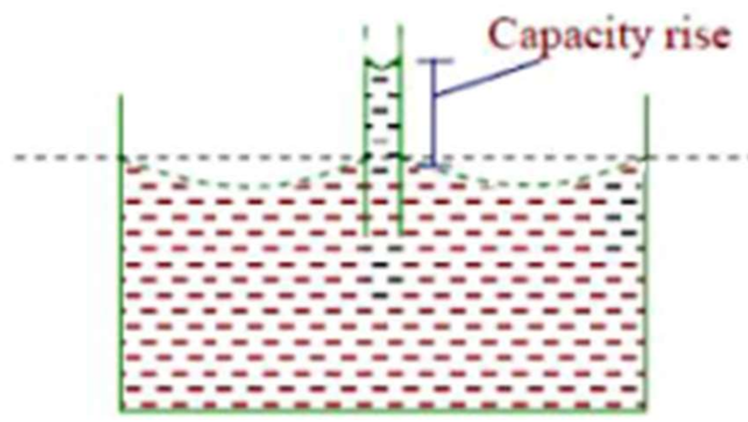
- The phenomenon of surface tension arises due to the two kinds of intermolecular forces
- Cohesion : The force of attraction between the molecules of a liquid by virtue of which they are bound to each other to remain as one assemblage of particles is known as the force of cohesion. This property enables the liquid to resist tensile stress.
- Adhesion : The force of attraction between unlike molecules, i.e. between the molecules of different liquids or between the molecules of a liquid and those of a solid body when they are in contact with each other, is known as the force of adhesion. This force enables two different liquids to adhere to each other or a liquid to adhere to a solid body or surface.



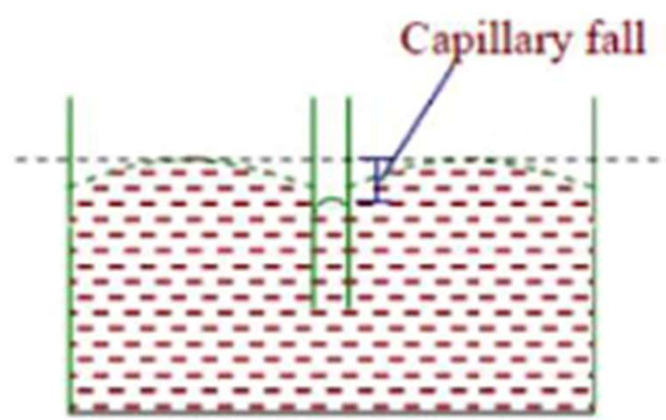
Capillarity

- Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them.
- Capillarity is due to cohesion adhesion and surface tension of liquids.
- If adhesion is more than cohesion then there will be capillary rise.
- If cohesion is greater than adhesion then will be capillary fall or depression.

$$h = \frac{2 \times T \times cos\theta}{\rho \times g \times r}$$



Cohesion < Adhesion



Cohesion > Adhesion

COMPRESSIBILITY AND BULK MODULUS

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Let $\forall$ = Volume of a gas enclosed in the cylinder
p = Pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

Then increase in pressure $= dp$ kgf/m^2

Decrease in volume $= d\forall$

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$

– ve sign means the volume decreases with increase of pressure.

$$\therefore \text{ Bulk modulus } \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

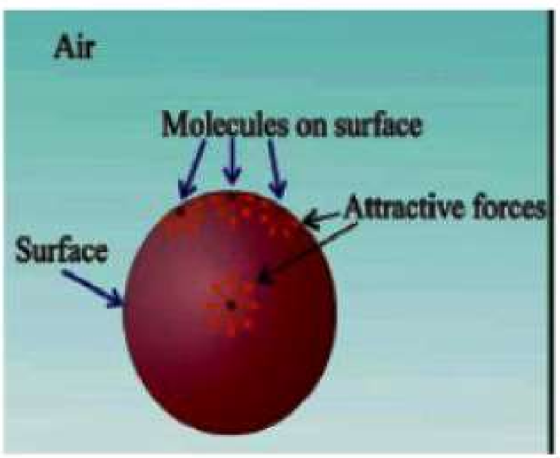
$$= \frac{dp}{\frac{-d\forall}{\forall}} = \frac{-dp}{d\forall}\forall$$

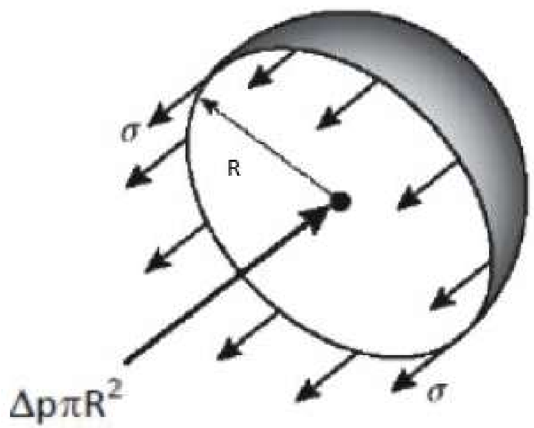
$$\text{Compressibility} = \frac{1}{K}$$

Pressure inside a Water Droplet

- Liquid droplets tend to assume a spherical shape since a sphere has the smallest surface area per unit volume.

The force developed around the edge of the cut sphere is 2R. This force must be balance with the difference between the internal pressure P and the external pressure p acting on the circular area of the cut.



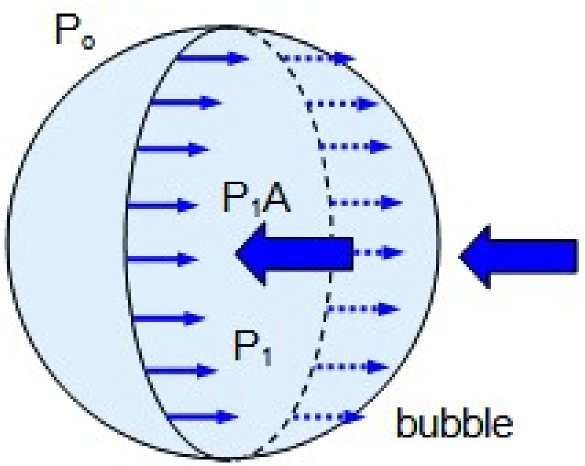


$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = \frac{2\sigma}{R}$$

Pressure within a Soap bubble

- The fact that air has to be blown into a drop of soap solution to make a bubble should suggest that the pressure within the bubble is greater than that outside.
- This is in fact the case this excess pressure creates a force that is just balanced by the inward pull of the soap film of the bubble due to its surface tension.



$$2 \times 2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = \frac{4\sigma}{R}$$

Surface Tension on a Liquid Jet.

Let p = Pressure intensity inside the liquid jet above the outside pressure
σ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

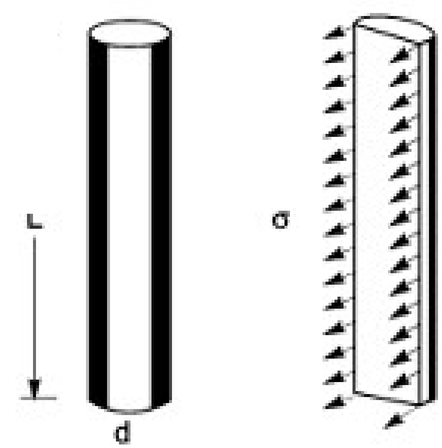
Force due to pressure $= p \times$ area of semi jet
$= p \times L \times d$

Force due to surface tension $= \sigma \times 2L$.

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

$$\therefore \quad p = \frac{\sigma \times 2L}{L \times d}$$



Example problems

Calculate the density, specific wt and specific gravity of one liter of liquid which weighs 7 N

Given : Weight = 7 N

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3$$

$$\left(\because \; 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \; \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

(*i*) $$\text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7\,\text{N}}{\left(\frac{1}{1000}\right)\text{m}^3} = \mathbf{7000\ N/m^3}.$$

(*ii*) $$\text{Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5\ kg/m^3}.$$

(*iii*) $$\text{Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} = \mathbf{0.7135}.$$

$\{\because$ Density of water = 1000 kg/m$^3\}$

Example problems

Calculate the density specific wt and weight of one liter of petrol with specific gravity 0.7

Given : Volume = 1 litre = $1 \times 1000\ \text{cm}^3$

$$= \frac{1000}{10^6}\ \text{m}^3 = 0.001\ \text{m}^3$$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

The density of a liquid $= S \times \text{Density of water}$

Density (ρ) $= S \times 1000\ \text{kg/m}^3 = 0.7 \times 1000 = \mathbf{700\ kg/m^3}$.

(*ii*) *Specific weight* (*w*)

$$w = \rho \times g = 700 \times 9.81\ \text{N/m}^3 = \mathbf{6867\ N/m^3}.$$

(*iii*) *Weight (W)*

We know that specific weight $= \dfrac{\text{Weight}}{\text{Volume}}$

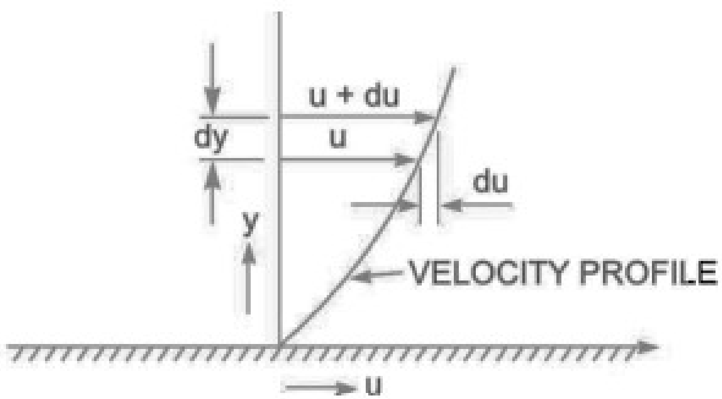
or $w = \dfrac{W}{0.001}$ or $6867 = \dfrac{W}{0.001}$

∴ $W = 6867 \times 0.001 = \mathbf{6.867\ N}$.

Understanding Viscosity

$$\tau = \mu \frac{du}{dy}.$$

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to *y*



Velocity variation near a solid boundary.

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If the velocity distribution over a plate is given by u = (2/3) y – y^2 in which u is the velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Given :

$$u = \frac{2}{3} y - y^2$$

$$\mu = 8.63 \text{ poise}$$

$$\tau = \mu \frac{du}{dy}.$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

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Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863 \text{ N s/m}^2$

$$\tau = \mu \frac{du}{dy}.$$

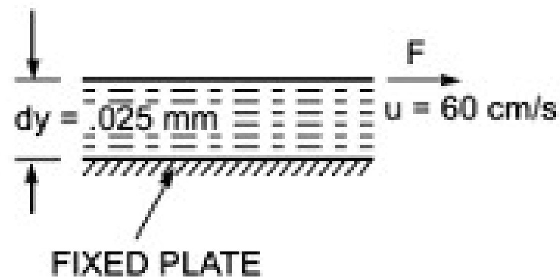
(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756 \text{ N/m}^2}.$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167 \text{ N/m}^2}.$$

A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m^2 to maintain this speed. Determine the fluid viscosity between the plates.



Given :

Distance between plates, $dy = .025 \text{ mm} = .025 \times 10^{-3} \text{ m}$

Velocity of upper plate, $u = 60 \text{ cm/s} = 0.6 \text{ m/s}$

Force on upper plate, $F = 2.0 \dfrac{\text{N}}{\text{m}^2}$.

$$\tau = \mu \frac{du}{dy}.$$

du = Change of velocity = $u - 0 = u = 0.60$ m/s

dy = Change of distance = $.025 \times 10^{-3}$ m

τ = Force per unit area = $2.0 \dfrac{\text{N}}{\text{m}^2}$

$$2.0 = \mu \frac{0.60}{.025 \times 10^{-3}}$$

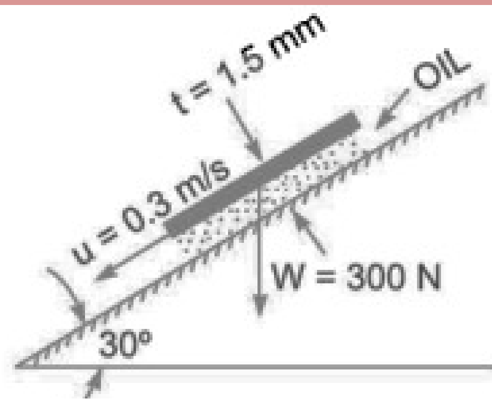
$$\therefore \quad \mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60}$$

$$= 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$$

$$= 8.33 \times 10^{-5} \times 10 \text{ poise}$$

$$= \mathbf{8.33 \times 10^{-4} \text{ poise.}}$$

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m X 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.



Given :

Area of plate,	$A = 0.8 \times 0.8 = 0.64 \text{ m}^2$
Angle of plane,	$\theta = 30°$
Weight of plate,	$W = 300$ N
Velocity of plate,	$u = 0.3$ m/s
Thickness of oil film,	$t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m

Component of weight W, along the plane = $W \cos 60° = 300 \cos 60° = 150$ N

shear force, F , on the bottom surface of the plate = 150 N

shear stress, $\tau = \frac{F}{\text{Area}}$

$$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

$$\tau = \mu \frac{du}{dy}$$

du = change of velocity = $u - 0 = u = 0.3$ m/s

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \textbf{11.7 poise.}$$

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Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9

Kinematic viscosity $\nu = 6 \text{ stokes} = 6 \text{ cm}^2/\text{s} = 6 \times 10^{-4} \text{ m}^2/\text{s}$

Sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

Now sp. gr. of a liquid $= \dfrac{\text{Density of the liquid}}{\text{Density of water}}$

or $$1.9 = \frac{\text{Density of liquid}}{1000}$$

$\therefore$ Density of liquid $= 1000 \times 1.9 = 1900 \dfrac{\text{kg}}{\text{m}^3}$

$\therefore$ Using the relation $\nu = \dfrac{\mu}{\rho}$, we get

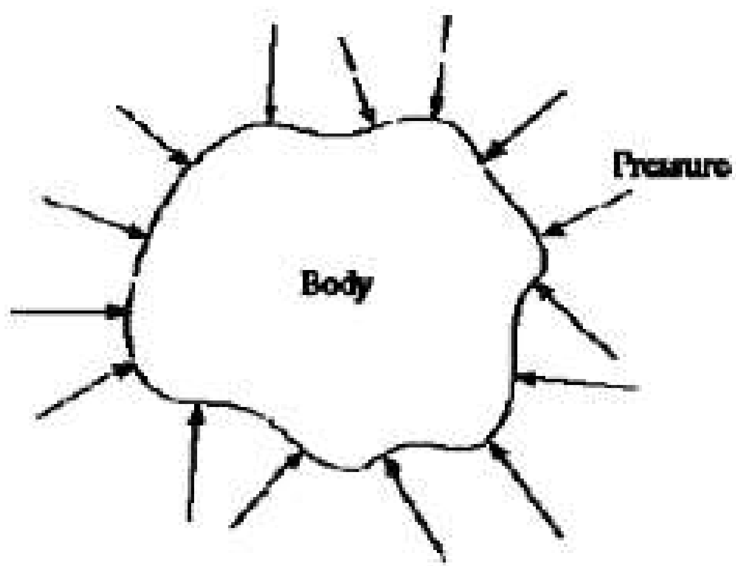
$$6 \times 10^{-4} = \frac{\mu}{1900}$$

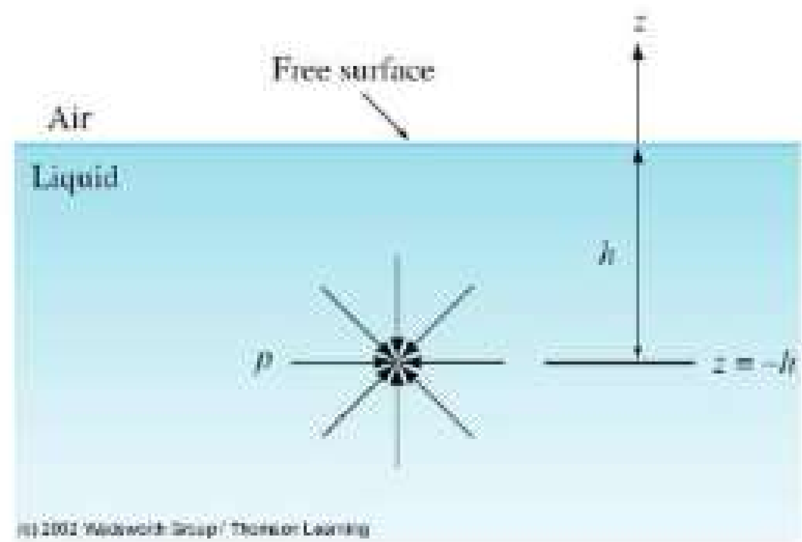
or $$\mu = 6 \times 10^{-4} \times 1900 = 1.14 \text{ Ns/m}^2$$
$$= 1.14 \times 10 = \mathbf{11.40 \text{ poise. Ans.}}$$

Fluid Pressure and Its Measurements

Fluid Pressure and Its Measurements

- When a certain mass of fluids is held in static equilibrium by conning it within solid boundaries, it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure





Definition of Pressure

- Pressure is one of the basic properties of all fluids. Pressure (p) is the force (F) exerted on or by the fluid on a unit of surface area (A).

$$p = \frac{F}{A}$$

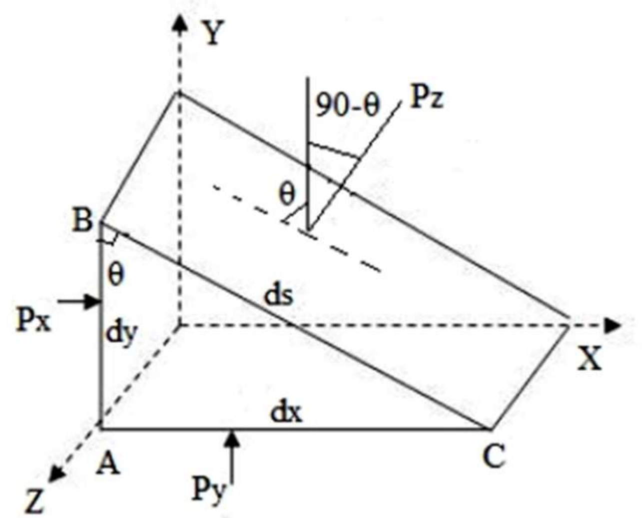
The basic unit of pressure is Pascal (Pa). When a fluid exerts a force of 1 N over an area of 1 m^2 , the pressure equals one Pascal, i.e., 1 Pa = 1 N/ m^2

Pressure at a Point and Pascal's Law

- Pascal's Principle: According to Pascal's Law, Pressure or intensity of pressure at a point in a static fluid will be equal in all directions.
- By considering the equilibrium of a small triangular wedge of fluid extracted from a static fluid body, one can show that for any wedge angle , the pressures on the three faces of the wedge are equal in magnitude.

Derivation

Let us consider one arbitrary fluid element of triangular shape ABC as displayed here in following figure. Let us assume that width of fluid element ABC perpendicular to the plane of paper is unity.



Let us consider the following terms as mentioned here:

P_x = Pressure acting in X- direction over the face AB

P_y= Pressure acting in Y- direction over the face AC

P_z= Pressure acting in Z- direction over the face BC

θ = Angle ABC, as displayed above in figure

dx, dy and ds be Fluid element dimensions

ρ= Density of the fluid

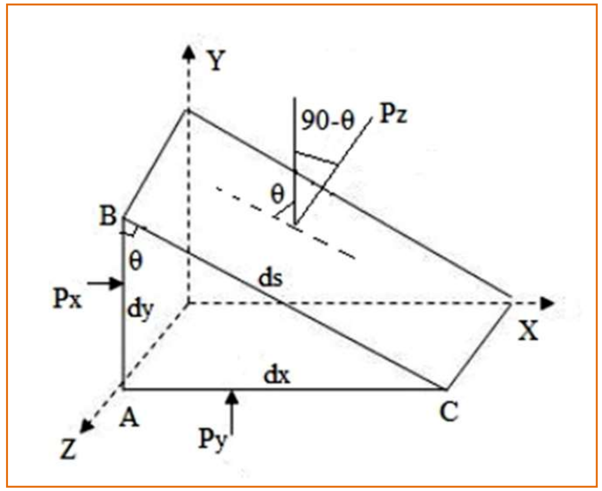
Let us analyze here the forces acting on the fluid element ABC

Force on the face AB, AC and BC

$$F_{AB} = P_X \times Area\ of\ face\ AB = P_X.dy.1 = P_X.dy$$

$$F_{AC} = P_Y \times Area\ of\ face\ AC = P_Y.dx.1 = P_Y.dx$$

$$F_{BC} = P_Z \times Area\ of\ face\ BC = P_Z.ds.1 = P_Z.ds$$



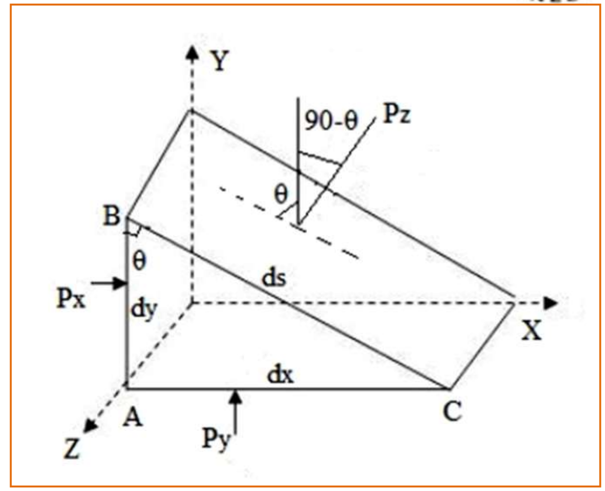
Weight of the fluid element,

$$W = Volume \times Density\ of\ fluid \times acceleration\ due\ to\ gravity$$

$$W = Area \times width of fluid element \times Density\ of\ fluid\ \times acceleration\ due\ to\ gravity$$

$$W = \frac{AB \times AC}{2} \times 1 \times \rho \times g = \frac{dydx}{2} \times \rho \times g$$

resolving the forces in X-direction



$$P_Y.dx - P_Z.dsSin(90-\theta) = 0$$

$$P_X.dy = P_Z.dsCos\theta$$

As we can see from above fluid element ABC, dy = ds Cosθ

$$P_X.dy = P_Z.dy \quad (1)$$

$$\implies P_X = P_Z$$

Resolving the forces in Y-direction,

$$P_Y.dx - P_Z.dsCos(90-\theta) - (dydx/2) \times \rho \times g = 0$$

$$P_Y.dx - P_Z.dsSin\theta - (dy\ dx/2) \times \rho \times g = 0$$

As fluid element is very small and therefore, we can neglect the weight of fluid element

$$P_Y.dx - P_Z.dsSin\theta = 0$$

As we can see from above fluid element ABC, dx = ds Sin θ

$$P_Y.dx - P_Z.dx = 0$$

$$P_Y = P_Z \qquad (2)$$

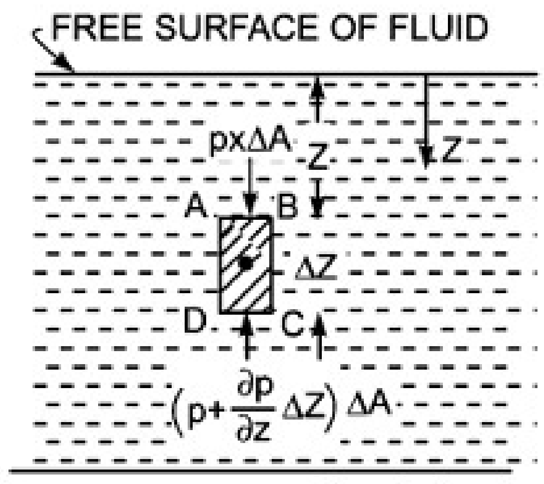
From above two expressions (1) and (2), we can write following equation as mentioned here,

$$P_X = P_Y = P_Z$$

Pressure Variation with Depth

Hydrostatic Law: rate of increase of pressure in a vertically downward direction in a fluid must be equal to specific wt. of the liquid at that point

Consider a small fluid element as shown in Fig.



Let ΔA= Cross-sectional area of element

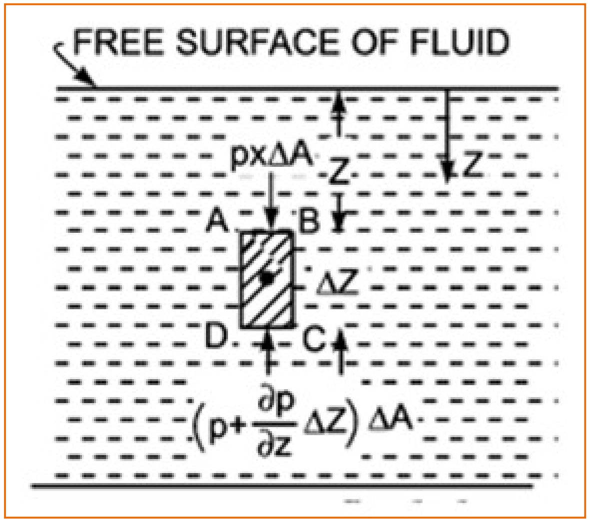
ΔZ = Height of fluid element

p = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are :

1. Pressure force on $AB = p \times \Delta A$ and acting perpendicular to face AB in the downward direction.
2. Pressure force on $CD = \left(p + \frac{\partial p}{\partial Z}\Delta Z\right) \times \Delta A$, acting perpendicular to face CD, vertically upward direction.
3. Weight of fluid element = Density × g × Volume = $\rho \times g \times (\Delta A \times \Delta Z)$.
4. Pressure forces on surfaces BC and AD are equal and opposite.



For equilibrium of fluid we have

$$p\Delta A - \left(p + \frac{\partial p}{\partial Z}\Delta Z\right)\Delta A + \rho \times g \times (\Delta A \times \Delta Z) = 0$$

or $$p\Delta A - p\Delta A - \frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A \times Z = 0$$

or $$-\frac{\partial p}{\partial Z}\Delta Z\Delta A + \rho \times g \times \Delta A\Delta Z = 0$$

or $$\frac{\partial p}{\partial Z}\Delta Z\Delta A = \rho \times g \times \Delta A\Delta Z \quad \text{or} \quad \frac{\partial p}{\partial Z} = \rho \times g \text{ [cancelling } \Delta A\Delta Z \text{ on both sides]}$$

∴ $$\frac{\partial p}{\partial Z} = \rho \times g = w \qquad (\because \rho \times g = w)$$

By integrating the above equation for liquids, we get

$$\int dp = \int \rho g dZ$$

or

$$p = \rho g Z$$

or

$$Z = \frac{p}{\rho \times g}$$

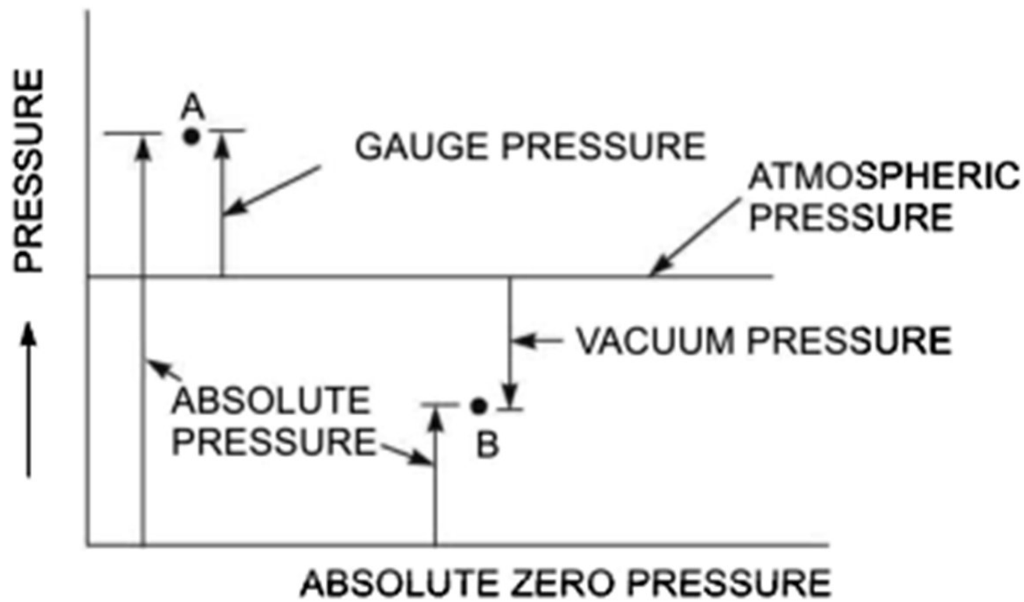
where p is the pressure above atmospheric pressure and Z is the height of the point from free surfaces.

Here Z is called **pressure head.**

Types of pressure

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

- Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.
- Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.
- Vacuum pressure is defined as the pressure below the atmospheric pressure



(*i*) Absolute pressure = Atmospheric pressure + Gauge pressure

$$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure = Atmospheric pressure – Absolute pressure.

Example problem

Calculate the pressure due to a column of 0.3 m of (a) water, (b) an oil of sp. gr. 0.8, and (c) mercury of sp. gr. 13.6. Take density of water, $\rho = 1000\ kg/m^3$

Height of liquid column, $Z = 0.3$ m.

$$p = \rho g Z$$

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(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

$\therefore$

$$p = \rho g Z = 1000 \times 9.81 \times 0.3 = 2943 \text{ N/m}^2$$

$$= \frac{2943}{10^4} \text{ N/cm}^2 = \mathbf{0.2943 \text{ N/cm}^2.}$$

(*b*) For oil of sp. gr. 0.8,

Density of oil, ρ_0 = Sp. gr. of oil × Density of water

(ρ_0 = Density of oil)

$$= 0.8 \times \rho = 0.8 \times 1000 = 800 \text{ kg/m}^3$$

Now pressure, $p = \rho_0 \times g \times Z$

$$= 800 \times 9.81 \times 0.3 = 2354.4 \frac{\text{N}}{\text{m}^2} = \frac{2354.4}{10^4} \frac{\text{N}}{\text{cm}^2}.$$

$$= \mathbf{0.2354 \frac{N}{cm^2}}.$$

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(*c*) For mercury, sp. gr. = 13.6

Density of mercury ρ_s = Specific gravity of mercury × Density of water

$$= 13.6 \times 1000 = 13600 \text{ kg/m}^3$$

$$\therefore \quad p = \rho_s \times g \times Z = 13600 \times 9.81 \times 0.3 = 40025 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{40025}{10^4} = \mathbf{4.002 \frac{N}{cm^2}}.$$

The pressure intensity at a point in a fluid is given 3.924N/cm². Find the corresponding height of fluid when the fluid is : (a) water, and (b) oil of sp. gr. 0.9.

Solution.

Given :

Pressure intensity, $p = 3.924 \frac{N}{cm^2} = 3.924 \times 10^4 \frac{N}{m^2}$.

$$Z = \frac{p}{\rho \times g}$$

(*a*) For water, $\rho = 1000 \text{ kg/m}^3$

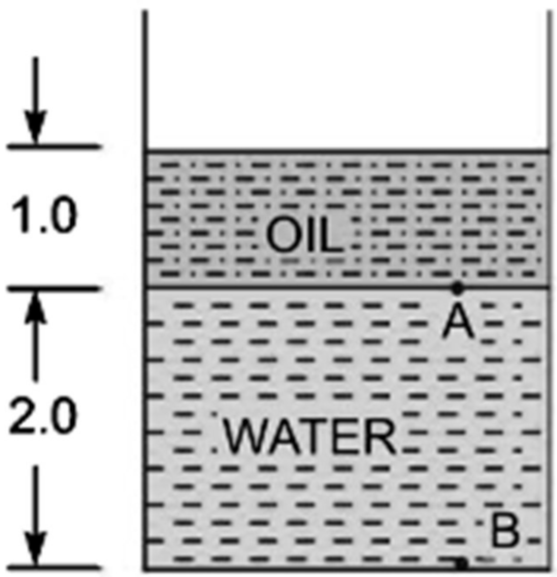
$$\therefore \quad Z = \frac{p}{\rho \times g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = \textbf{4 m of water.}$$

(*b*) For oil, sp. gr. $= 0.9$

$\therefore$ Density of oil $\rho_0 = 0.9 \times 1000 = 900 \text{ kg/m}^3$

$$\therefore \quad Z = \frac{p}{\rho_0 \times g} = \frac{3.924 \times 10^4}{900 \times 9.81} = \textbf{4.44 m of oil.}$$

An open tank contains water up to a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth of 1m. Find the pressure intensity (i) at the interface of the two liquids, and (ii) at the bottom of the tank.



Given :

Height of water, $Z_1 = 2$ m

Height of oil, $Z_2 = 1$ m

Sp. gr. of oil, $S_0 = 0.9$

Density of water, $\rho_1 = 1000$ kg/m^3

Density of oil, $\rho_2 =$ Sp. gr. of oil $\times$ Density of water

$= 0.9 \times 1000 = 900$ kg/m^3

Pressure intensity at any point is given by $p = \rho \times g \times Z.$

(*i*) At interface, *i.e.*, at *A* $p = \rho_2 \times g \times 1.0$

$$= 900 \times 9.81 \times 1.0$$

$$= 8829 \frac{\text{N}}{\text{m}^2} = \frac{8829}{10^4} = \mathbf{0.8829\ N/cm^2.}$$

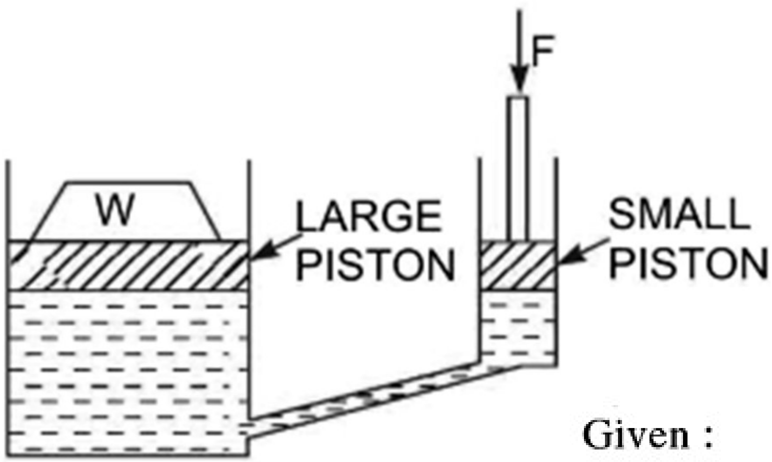
(*ii*) At the bottom, *i.e.*, at *B*

$$p = \rho_2 \times gZ_2 + \rho_1 \times g \times Z_1$$

$$= 900 \times 9.81 \times 1.0 + 1000 \times 9.81 \times 2.0$$

$$= 8829 + 19620 = 28449 \text{ N/m}^2 = \mathbf{2.8449\ N/cm^2.}$$

The diameters of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when: (a) the pistons are at the same level. (b) small piston is 40 cm above the large piston. The density of the liquid in the jack is given as 1000 kg/m^3.



Given :

Dia. of small piston, $d = 3$ cm

Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 \text{ cm}^2$

Dia. of large piston, $D = 10$ cm

$\therefore$ Area of larger piston, $A = \frac{\pi}{4} \times (10)^2 = 78.54 \text{ cm}^2$

Force on small piston, $F = 80$ N

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Let the load lifted $= W$.

(a) When the pistons are at the same level

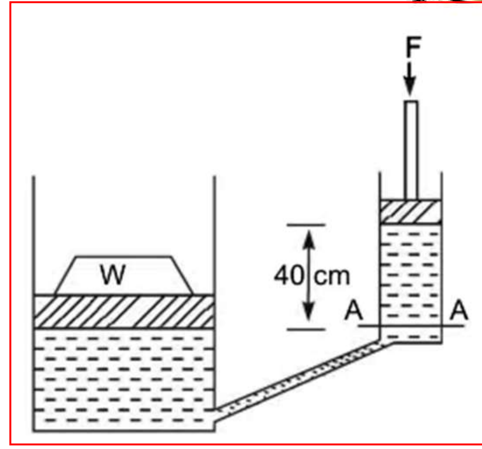
Pressure intensity on small piston $\frac{F}{a} = \frac{80}{7.068}$ N/cm^2

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston $= \frac{80}{7.068}$

$\therefore$ Force on the large piston $=$ Pressure $\times$ Area

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96\ N.}$$



(b) When the small piston is 40 cm above the large piston

Pressure intensity on the small piston $= \frac{F}{a} = \frac{80}{7.068} \frac{N}{cm^2}$

Pressure intensity at section A-A $= \frac{F}{a}$ + Pressure intensity due to height of 40 cm of liquid.

pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2 = \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

Pressure intensity at section A-A $= \frac{80}{7.068} + 0.3924 = 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$

$\therefore$ Pressure intensity transmitted to the large piston $= 11.71 \text{ N/cm}^2$

Force on the large piston = Pressure × Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3

Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

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Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg

= Sp. gr. of mercury × Density of water

$= 13.6 \times 1000 \text{ kg/m}^3$

Z_0 = Pressure head in terms of mercury.

$$p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \quad (\because \quad Z_0 = 0.75)$$
$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1 = (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

Gauge pressure, p = **45028 N/m²**.

absolute pressure = Gauge pressure + Atmospheric pressure

$$= 45028 + 100062 = \mathbf{145090 \text{ N/m}^2}.$$

Pressure measuring devices

The pressure of a fluid is measured by the following devices :

1. Manometers
2. Mechanical Gauges.

Manometers: Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(a) Simple Manometers,

(b) Differential Manometers.

Mechanical Gauges: Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are:

(a) Diaphragm pressure gauge,

(b) Bourdon tube pressure gauge,

(c) Dead-weight pressure gauge, and

(d) Bellows pressure gauge.

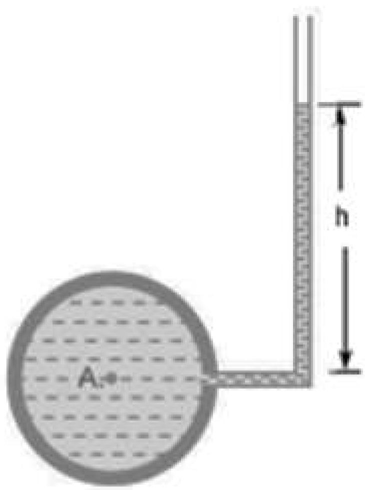
Simple manometer

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer
2. U-tube Manometer, and
3. Single Column Manometer.

Piezometer

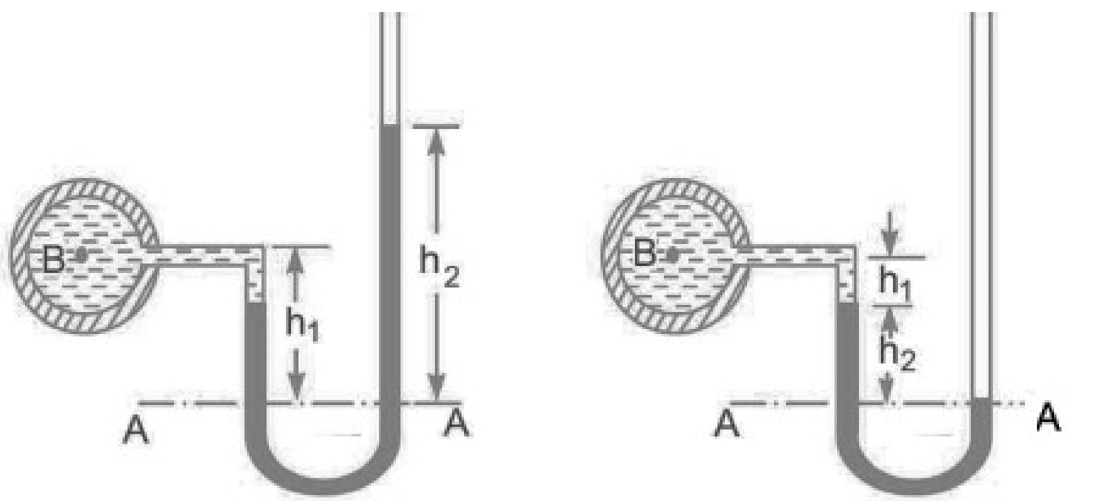
- One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere
- The rise of liquid gives the pressure head at that point.



$$\text{pressure at } A = \rho \times g \times h \ \frac{N}{m^2}.$$

U-tube Manometer

- Consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere
- The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured



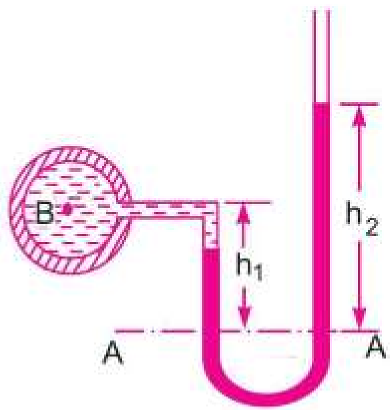
(a) For gauge pressure (b) For vacuum pressure

U-tube manometer……

For Gauge Pressure.

Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

- Let h_1 = Height of light liquid above the datum line
- h_2 = Height of heavy liquid above the datum line
- S_1 = Sp. gr. of light liquid
- P_1 = Density of light liquid = 1000 x S_1
- S_2 = Sp. gr. of heavy liquid
- P_2 = Density of heavy liquid = 1000 x S_2



(a) For gauge pressure

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column = p + P_1X g X h_1

Pressure above A-A in the right column = P_1 x g X h_1

Hence equating the two pressures p + P_1gh_1= $P2gh_2$

$$P = (P_2gh_2 - P_1g\,h_1)$$

U tube manometer……

For Vacuum Pressure.

For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in fig.

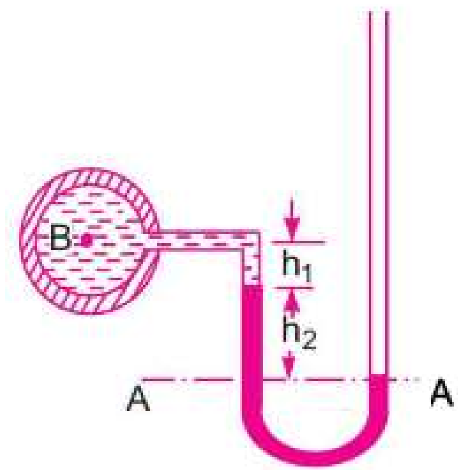
Then,

Pressure above A-A in the left column = $P_2gh_2 + P_1gh_1 + p$

Pressure head in the right column above A-A = 0

Therefore, $P_2gh_2 + P_1gh_1 + p = 0$

$$p = -(P_2gh_2 + P_1gh_1)$$

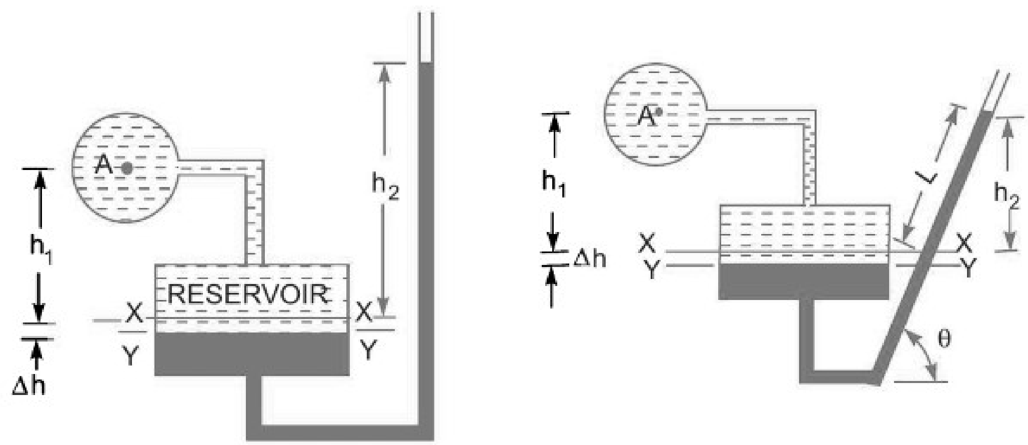


(b) For vacuum pressure

Single Column Manometer

- Single column manometer is a modified form of a U-tube manometer
- Has a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer
- Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb.

1. ***Vertical Single Column Manometer.***
2. ***Inclined Single Column Manometer.***



Pressure measurement in single column manometer

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

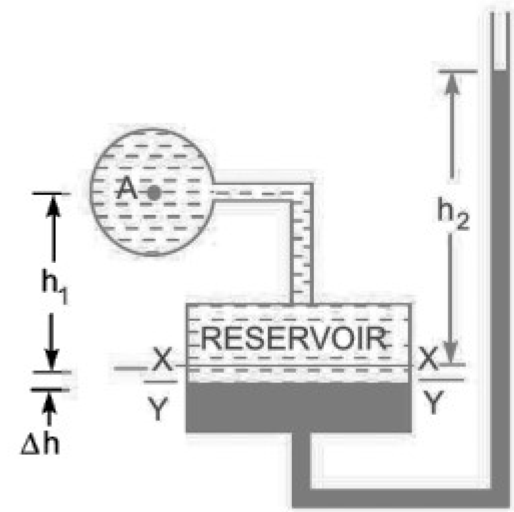
a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir



Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \quad \ldots(i)$$

Now consider the datum line Y-Y as shown in Fig. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y = $\rho_1 \times g \times (\Delta h + h_1) + p_A$

single column manometer……

Pressure in the left limb above Y-$Y = \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (i), $\Delta h = \dfrac{a \times h_2}{A}$

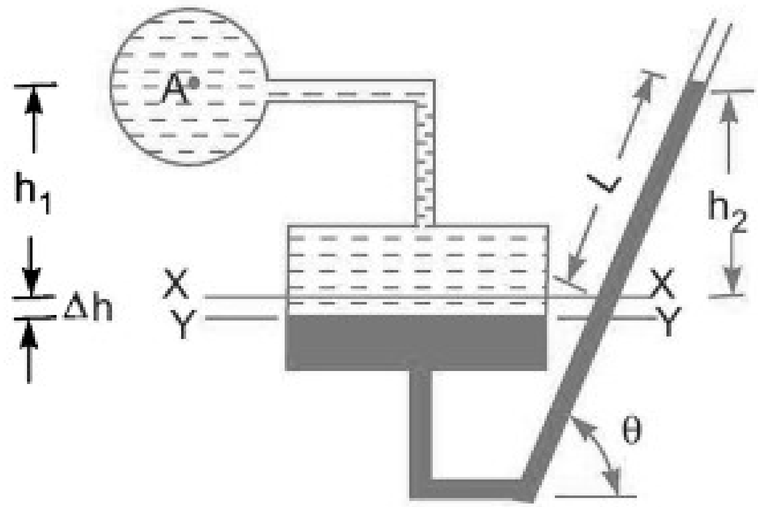
$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

As the area A is very large as compared to a, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then

$$p_A = h_2\rho_2 g - h_1\rho_1 g$$

Inclined Single Column Manometer



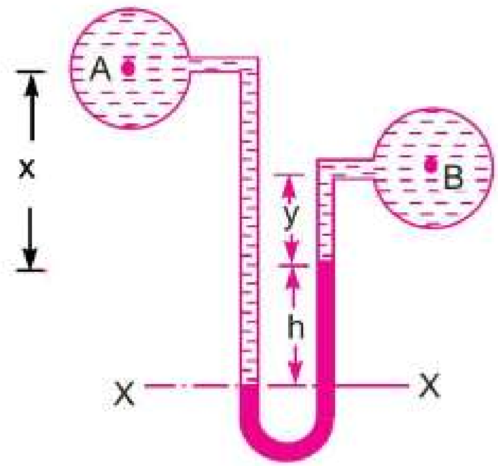
$$p_A = \sin\theta \times \rho_2 g - h_1 \rho_1 g.$$

DIFFERENTIAL MANOMETERS

- Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes.
- A differential manometer consists of a U'-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

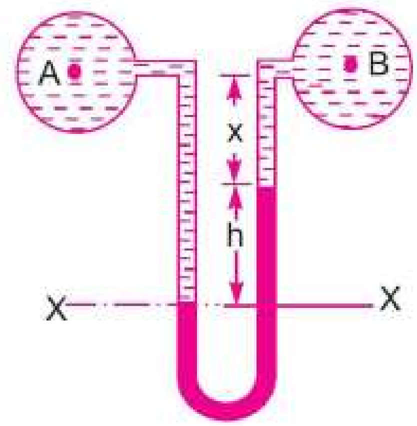
1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

DIFFERENTIAL MANOMETERS



(a)Two pipes at different levels

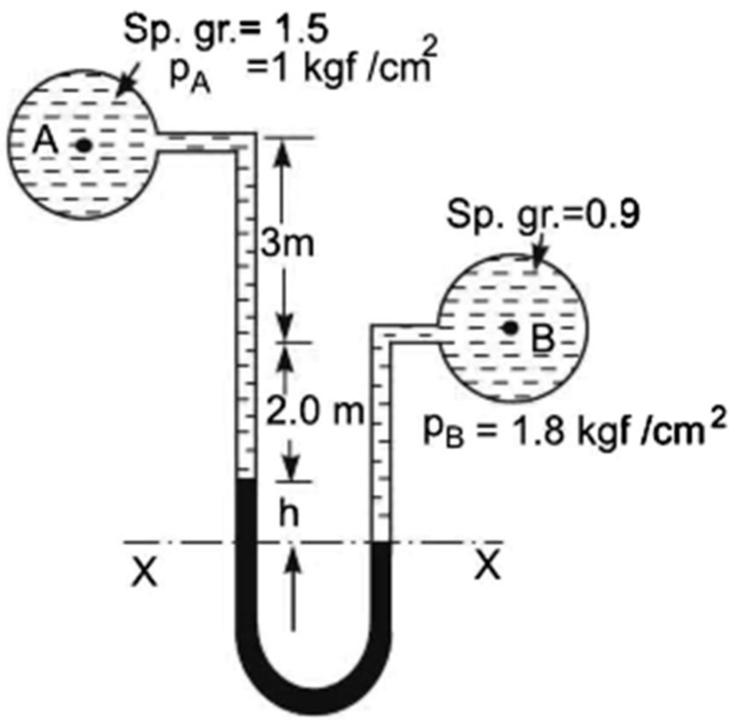
$$p_A - p_B = h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$



(b) A and B are at the same level

$$p_A - p_B = g \times h(\rho_g - \rho_1).$$

A differential manometer is connected at the two points A and B of two pipes as shown in Fig. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and Bare 1 kgf/cm^2 and 1.80 kgf/cm^2 respectively. Find the difference in mercury level in the differential manometer.



Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm^2 $= 1 \times 10^4$ kgf/m^2

$= 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm^2

$= 1.8 \times 10^4$ kgf/m^2

$= 1.8 \times 10^4 \times 9.81$ N/m^2 ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m^3

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

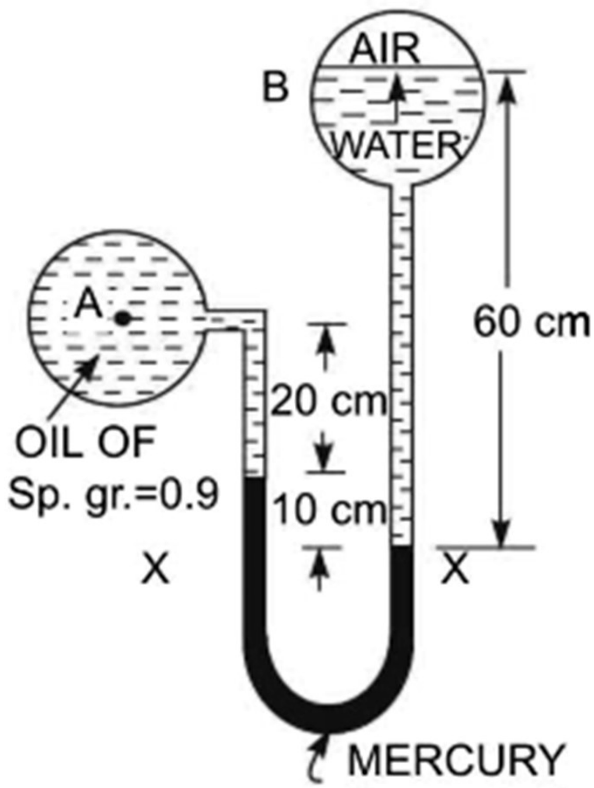
$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1\ cm.}$$

A differential manometer is connected at the two points A and B as shown in Fig. At B air pressure is 9.81 N/cm² (abs), find the absolute pressure at A.



Given :

Air pressure at $B = 9.81\ \text{N/cm}^2$

or $p_B = 9.81 \times 10^4\ \text{N/m}^2$

Density of oil $= 0.9 \times 1000 = 900\ \text{kg/m}^3$

Density of mercury $= 13.6 \times 1000\ \text{kg/m}^3$

Let the pressure at A is p_A

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Taking datum line at X-X

Pressure above X-X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B$$
$$= 5886 + 98100 = 103986$$

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$
$$= 13341.6 + 1765.8 + p_A$$

Equating the two pressure heads

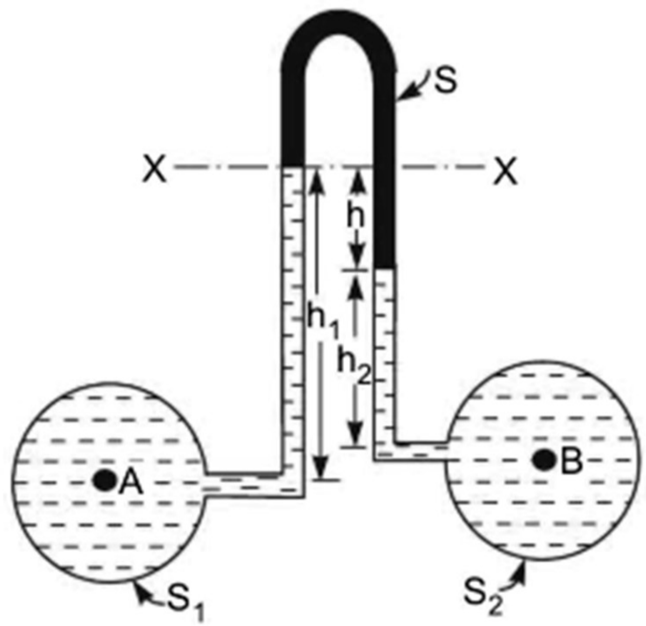
$$103986 = 13341.6 + 1765.8 + p_A$$

$$\therefore \quad p_A = 103986 - 15107.4 = 88876.8$$

$$\therefore \quad p_A = 88876.8 \text{ N/m}^2 = \frac{88876.8\text{N}}{10000 \text{ cm}^2} = 8.887 \frac{\text{N}}{\text{cm}^2}.$$

$\therefore$ Absolute pressure at A = **8.887 N/cm²**.

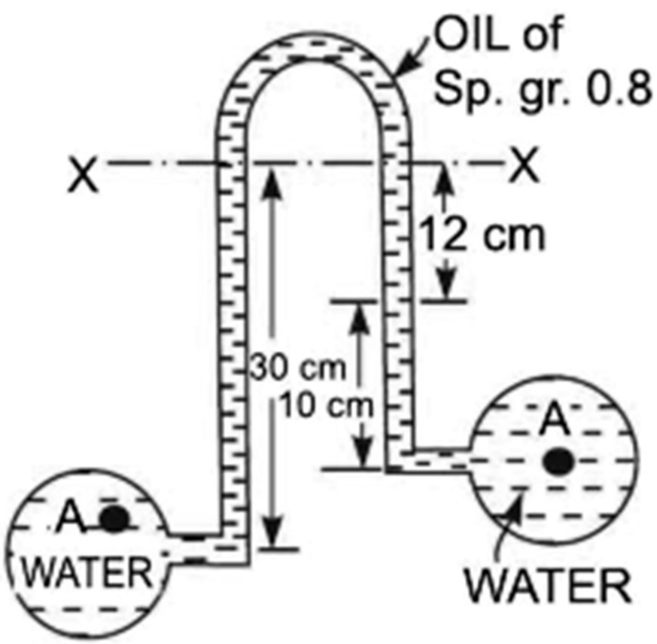
Inverted U-tube Differential Manometer.



$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig.



Given :

Pressure head at $A = \frac{p_A}{\rho g} = 2$ m of water

$\therefore \quad p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$

Fig. shows the arrangement.

Taking *X-X* as datum line.

Pressure below *X-X* in the left limb $= p_A - \rho_1 \times g \times h_1$

$$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$$

Pressure below *X-X* in the right limb

$$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

$$= p_B - 981 - 941.76 = p_B - 1922.76$$

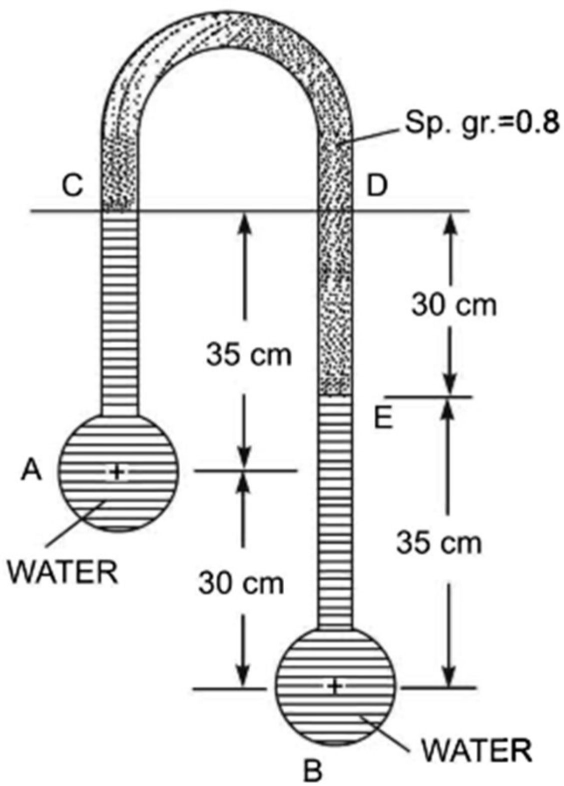
Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $$p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$$

or $$p_B = \mathbf{1.8599 \text{ N/cm}^2}.$$

An inverted U-tube manometer is connected to two horizontal pipes A and B through which water is flowing. The vertical distance between the axes of these pipes is 30 cm. When an oil of specific gravity 0.8 is used as a gauge fluid, the vertical heights of water columns in the two limbs of the inverted manometer (when measured from the respective centre lines of the pipes) are found to be same and equal to 35 cm. Determine the difference of pressure between the pipes.



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Let p_A = pressure at A

p_B = pressure at B.

The points C and D lie on the same horizontal line.

Hence pressure at C should be equal to pressure at D.

But pressure at C $= p_A - \rho g\, h$

$= p_A - 1000 \times 9.81 \times (0.35)$

And pressure at D $= p_B - \rho_1 g h_1 - \rho_2 g h_2$

$= p_B - 1000 \times 9.81 \times (0.35) - 800 \times 9.81 \times 0.3$

But pressure at C = pressure at D

$\therefore \quad p_A - 1000 \times 9.81 \times .35$

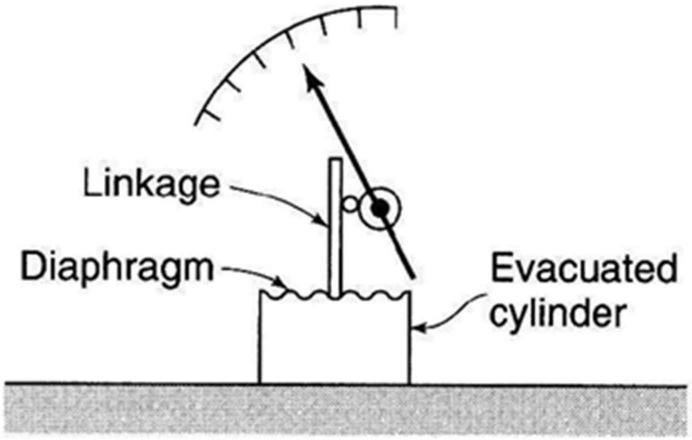
$= p_B - 1000 \times 9.81 \times 0.35 - 800 \times 9.81 \times 0.3$

or $\quad 800 \times 9.81 \times 0.3 = p_B - p_A$

or $$p_B - p_A = 800 \times 9.81 \times 0.3 = \mathbf{2354.4}\ \frac{\mathbf{N}}{\mathbf{m^2}}.$$

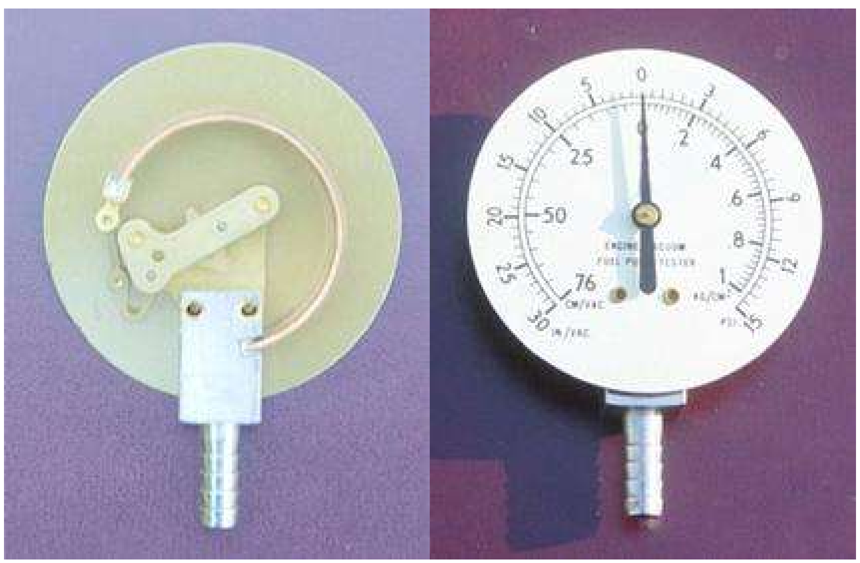
Mechanical and electronic pressure measuring devices

Aneroid Gauge



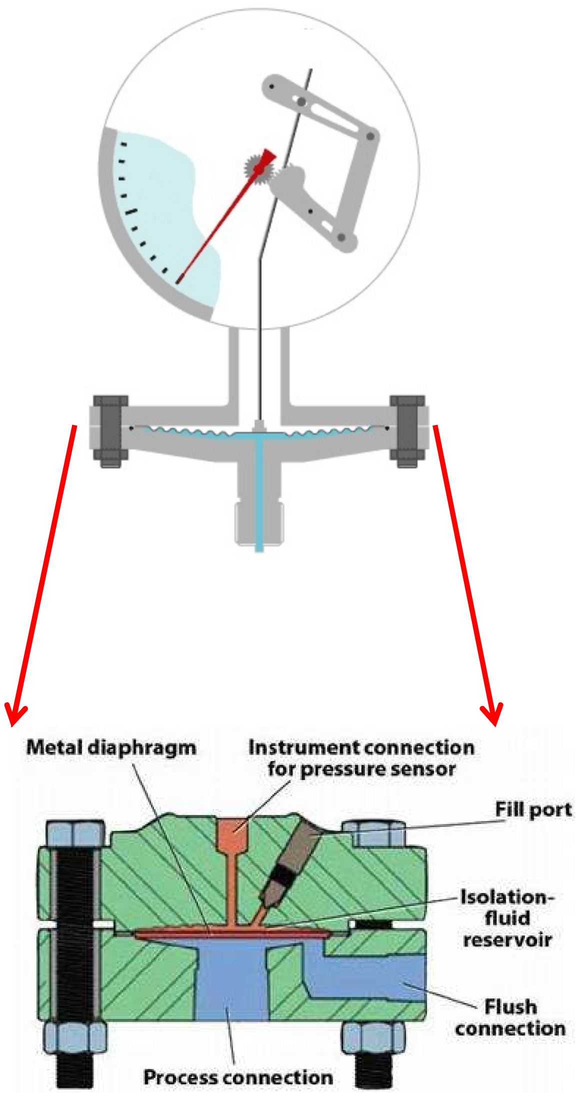
- Based on a metallic pressure-sensing element that flexes elastically under the effect of a pressure difference across the element
- The pressure sensing element may be a **Bourdon tube**, a diaphragm, a capsule, or a set of bellows, which will change shape in response to the pressure of the region in question. The deflection of the pressure sensing element may be read by a linkage connected to a needle, or it may be read by a secondary transducer.

Bourdon gauge



- It is used to measure high as well as low pressure.
- Pressure element consists of a metal tube of elliptical cross section. This tube is bend in a form of segment of circle and responds by bending inward due to increase in pressure
- When one end of tube is connected to source of pressure, the pressure inside the Bourdon tube causes the tube to expand and bend inward.
- A simple pinion and sector arrangement is provided to convert the linear movement of the tube into angular movement of the pointer.
- The pressure is indicated by the pointer over dial which can be graduated on a suitable scale.

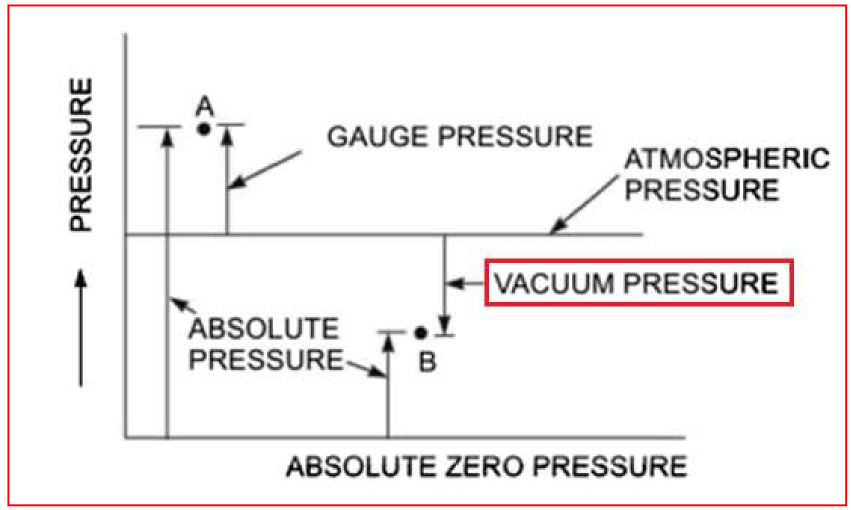
Diaphragm Gauge



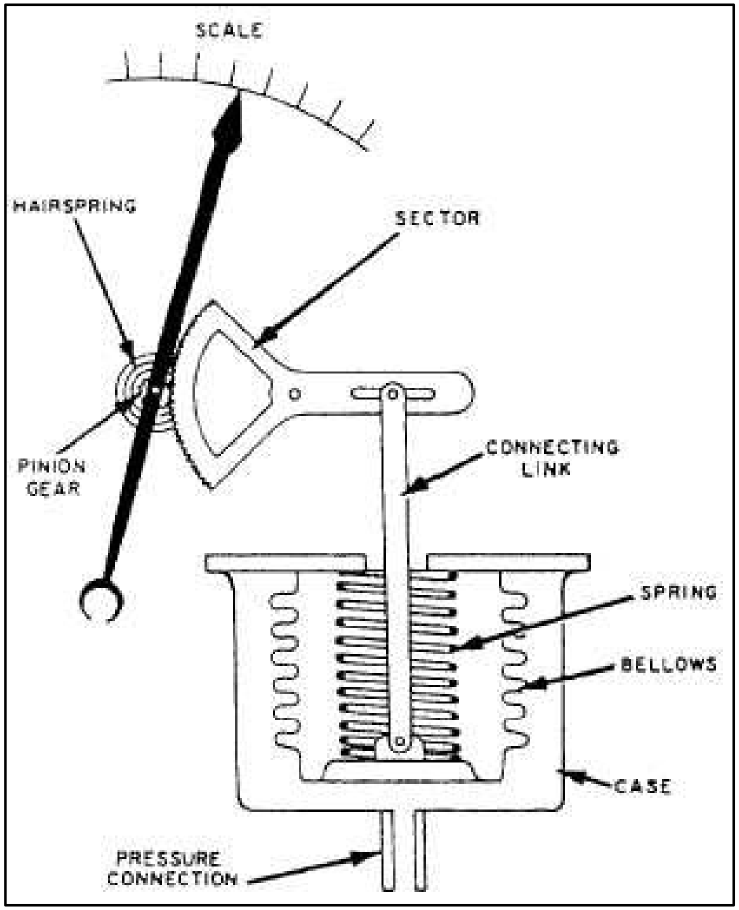
- Consists of metallic disc or diaphragm for actuating the pointer.
- When pressure is applied on lower side of diaphragm, it gets deflected upward.
- The movement of diaphragm is transmitted to a strain gauge or transducer which converts the pressure signal into electrical signal. In analogue devices, a rack and pinion system is provided which moves the pointer.

Vacuum gauge

- (a) Bourdon gauge can be used to measure vacuum by bending the tube inward instead of outward pressure in pressure gauge.
- (b)Vacuum gauge is graduated in mm of Hg below atmospheric pressure.

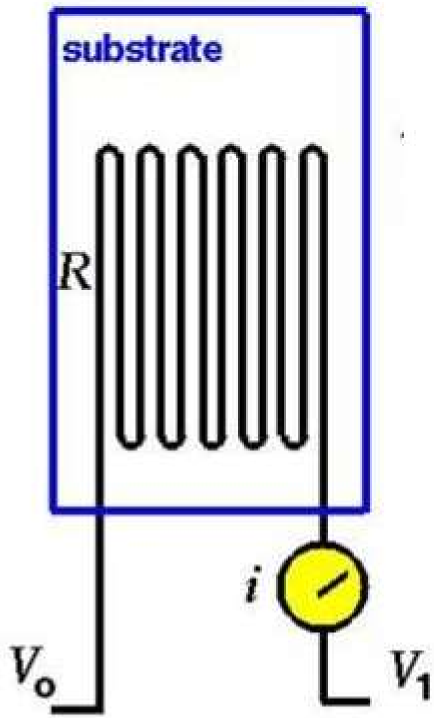


Bellows gauge



- A bellows gauge contains an elastic element that is a convoluted unit that expands and contracts axially with changes in pressure. The pressure to be measured can be applied to the outside or in side of the bellows.
- Most bellows gauges are spring-loaded; that is, a spring opposes the bellows, thu s preventing full expansion of the bellows.

Metal strain gauge



The strain gauge is generally glued (foil strain gauge) or deposited (thin-film strain gauge) onto a membrane. Membrane deflection due to pressure causes a resistance change in the strain gauge which can be electronically measured.

Stretching the wire causes the resistance to increase. During operation, a constant known voltage **V** is applied across the gage and the current **i** through the gage is measured

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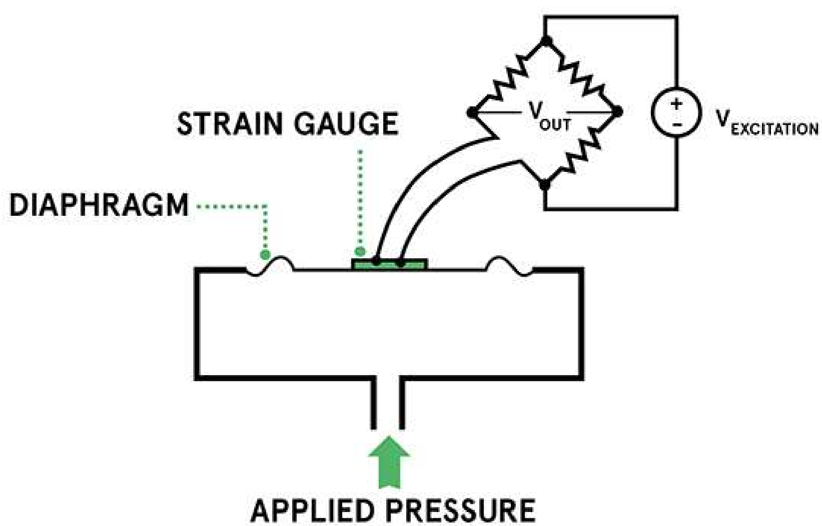
Piezo-resistive strain gauge



- The piezoresistive effect is a change in the electrical resistivity of a semiconductor or metal when mechanical strain is applied

- The basic principle of the piezoresistive pressure sensor is to use a strain gauge made from a conductive material that changes its electrical resistance when it is stretched. The strain gauge can be attached to a diaphragm that recognises a change in resistance when the sensor element is deformed. The change in resistance is converted to an output signal

Others..

- Capacitive Gauge: Uses a diaphragm and pressure cavity to create a variable capacitor to detect strain due to applied pressure.
- Magnetic pressure Gauge: Measures the displacement of a diaphragm by means of changes in inductance (reluctance), LVDT, Hall effect, or by eddy current principle.
- Piezoelectric pressure Gauge: Uses the piezoelectric effect in certain materials such as quartz to measure the strain upon the sensing mechanism due to pressure.
- Optical pressure Gauge: Uses the physical change of an optical fiber to detect strain due to applied pressure.
- Potentiometric pressure Gauge: Uses the motion of a wiper along a resistive mechanism to detect the strain caused by applied pressure.
- Resonant pressure Gauge: Uses the changes in resonant frequency in a sensing mechanism to measure stress, or changes in gas density, caused by applied pressure.