Module – 1 Introduction

Contents:-

1.1 Introduction

Structure is an assemblage of various structural components which resists deformation caused due to external loads. Structures are the means of transferring forces and moments.

Structure can be classified based on

- Size
- Shape or geometry
- Type of material
- Methods adopted for analysis and design.

The design of structure mainly includes

- Functional planning
- Structural design

Firstly, structure should serve the purpose for which it is intended and this can be achieved by proper functional planning which are carried out by architects.
Secondly, structure should have adequate strength in order to resist deformations caused due to external loads. Therefore, the structural designer should consider

1) The forces those are likely to act on the structure,

2) Should estimate the strength of the material,

3) Should design the cross-sectional dimensions of the structural components and

4) Should select the design methods that are adopted for design a structure.

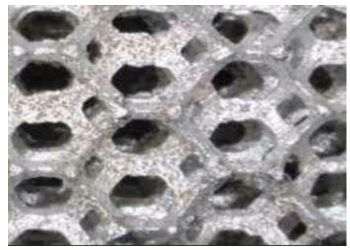
Steel structures are nothing but the assemblage of various structural steel components that are effectively joined or fastened to act as a single unit. When structural steel is used as a material for structures, structural design is known as **"Design of Steel Structures".**

1.2 Objective

- To know the advantages and disadvantages of steel over other construction materials
- To know different types of structural steel products and their applications
- To understand the behavior of mild steel under stress.

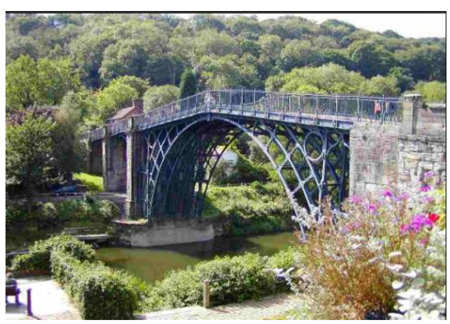
1.3 Historical Background

- Steel has been known since 3000BC.
- Ashoka pillar at Saranath was made with steel and iron joints are used in Puri Jaganath temples which are more than 1500 years old.
- In between 400-500BC, Foam steel was used in China & Europe.

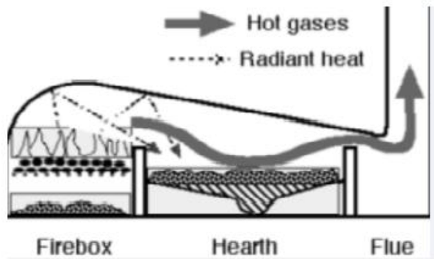
Note:- A metal foam is a cellular structure consists of solid metal aluminum or steel with gas filled pores comprising a large portion of volume which has low density and greater energy dissipation.

- Large scale use of iron for structural purpose was started in Europe in the later part of 18^{th} century.
- Cast iron was obtained from its ore by melting it in furnace fired by charcoal.
- The first major application of cast iron was in 30.4m span Coalbroakadale Arch Bridge which was designed by Abraham Darby and it was constructed across the river Severn at England in the year 1779.



- Later Darby found out a way of converting coal to coke, which made some changes in iron making process.
- A later development in the process was the combination of limestone with impurities of ore and coke to form slag. The product obtained from this process was very brittle in nature and liable to cracks under strain.
- Due to this disadvantage, a person named Henry Cort invented a reverberatory furnace in the year 1784. This method reduces the carbon content in steel and they named the

product as Wrought iron which was stronger, flexible and has high tensile strength compared to cast iron.



Reverberatory Furnace

- Carbon content in case of cast iron may vary in the range of 2 to 4% and in case of wrought iron; it is less than 0.15%.

- During the year 1826, wrought iron chains were used in Menai Strait Suspension Bridge constructed across Menai Strait river which was designed by Thomas Telford.

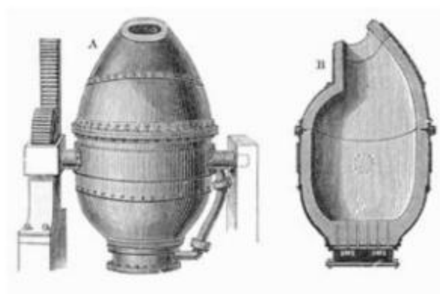
The Menai Suspension Bridge from a viewpoint on the A5 near the Britannia Bridge

- Britannia Bridge was the first Box- girder bridge made out of wrought iron material designed by Robert Stephenson and it was constructed across Menai Strait river in the year 1850.

Britannia Bridge
Pont Britannia

The modern Britannia Bridge.

- Structural steel was introduced in the year 1740, but it was not available in large quantities. Until Sir Henry Bessemer of England, invented and patented the iron making process in the year 1855.



Bessemer converter, schematic diagram

- In the year 1865, Siemens and Martin invented an open-hearth process which was extensively used for manufacture of structural steel.

- Carbon content in case of structural steel may vary at about 0.25 to 1.5%.
- The first major application of structural steel was Fowler and Baker's Forth Railway Bridge which was constructed across Fifth of Fourth river at Scotland in the year 1890.

1.4 Steel as construction material

Steel is an alloy of iron and other elements primarily carbon, nickel, manganese, silicon, molybdenum etc., which is widely used material for building structures because of various properties such as,

- **High strength to weight ratio**:- Steel is 10 times stronger than concrete. Because of high strength to weight ratio, slender sections can carry heavy loads with possible reduction of dead load. Due to this reason, steel has become an affordable material which can be used for making long span structures like auditoriums, railway bridges (eg:- Howrah Bridge in Calcutta), high rise buildings (eg:- Eiffel Tower in Paris), electric transmission towers, industrial buildings, tanks, chimneys etc.
- **Good ductility**
- **Great stiffness**:- Steel deforms very less to external loads. Steel does not require any special considerations.
- **Easy fabrication**:- Steel allows itself to be worked easily in fabricating shops in various ways like bending, sawing, flame cutting and welding.
- **Low cost**:- Cheaper than other materials. Aluminum cost about 3 to 4 times than usual grade of steel.
- **Easy Errection**:- No need of formwork for erecting steel structures.

1.5 Structural steel

It is a category of steel which is used as a construction material for manufacturing structural steel shapes like beams, plates, channel sections, hallow sections, angles etc.

Types of structural steel

- **Carbon steel**:- Carbon and manganese are the two elements imparting strength to this steel.

Minimum yield strength f_y = 230 to 300MPa

Minimum tensile strength f_u = 230 to 300MPa

- **High strength carbon steel**:- If the carbon content increases in steel, strength increases but ductility, toughness and weldability of steel reduces. This type of steel is employed in transmission lines and microwave towers.

Minimum yield strength f_y = 350 to 400MPa

Minimum tensile strength f_u = 480 to 550MPa

- **Medium and high strength micro-alloyed steel**:- Even though the carbon content is low, strength is mainly due to additional micro-alloy like titanium, boron (up to 0.25%).

Minimum yield strength f_y = 550 to 700MPa

Minimum tensile strength f_u = 440 to 590MPa

- **High strength quenched and tempered steel**:- Steel was heated in order to develop high strength. Even though it tough, it requires some welding techniques.

Minimum yield strength f_y = 550 to 700MPa

Minimum tensile strength f_u = 700 to 950MPa

- **Weathering steel**:- These are low-alloy, corrosive resistant steels. Therefore no need to paint the surfaces.

Minimum yield strength f_y = 350MPa

Minimum tensile strength f_u = 480MPa

1.6 Properties of structural steel

Physical properties

Property	Value
1. Unit mass / density of steel (ρ)	7850 kg/m^3
2. Modulus of elasticity (E)	2 X 10^{5}N/mm^2

3. Poisons ratio (μ)	0.3
4. Modulus of rigidity (G)	$0.769 \times 10^5 N/mm^2$
5. Co-efficient of thermal expansion (α)	$12 \times 10^{-6}/ °C$

Density:- defined as the ratio of mass by volume expressed in kg/m^3 or g/cm^3.

Modulus of elasticity:- defined as the ratio stress to strain within elastic limit also know as young's modulus.

Poisons ratio:- defined as the ratio of lateral strain to axial strain.

Modulus of rigidity:- defined as the ratio of shear stress to shear strain and also called as shear modulus.

Co-efficient of thermal expansion:- defined as the ratio of unit change in length or volume to the unit change in temperature and expressed in F or °C.

Mechanical properties

Property	**Value**
1. Yield stress (f_y)	220-450 N/mm^2
2. Ultimate or Tensile strength (f_u)	1.2 f_y
3. Percentage elongation	20
4. Notch toughness	-

Yield strength or yield stress:- defined as the minimum force required to cause permanent deformation in steel.

Ultimate or Tensile strength:- defined as the point at which permanent deformation takes place when the material is stretched or pulled along its length.

Percentage elongation:- defined as the ratio of increase in gauge length to original length expressed in percentage.

Notch toughness:- it is the ability of the material to absorb energy during rupture or fracture with the presence of notch or neck or flaws.

Other mechanical properties are

BHN (Brinell Hardness Number) = 150-190

VHN (Vickers Hardness Number) = 157-190

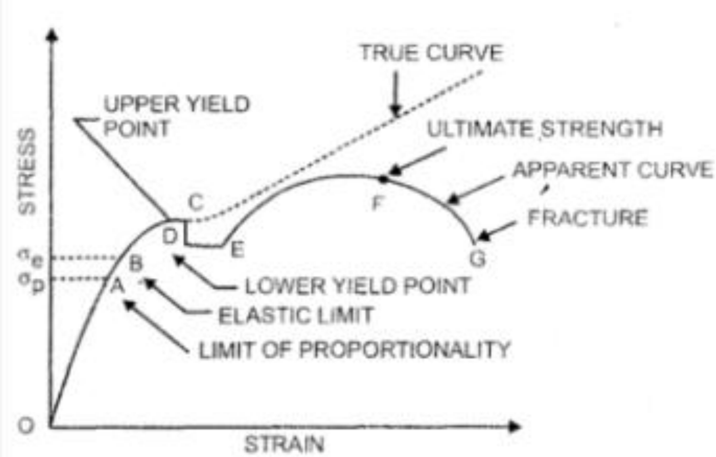
Approximate melting point = 1500°C

Thermal conductivity = 0.14cal/cm^2

Chemical composition of structural steel

Grade	Designation	Carbon	Manganese	Sulphur	Phosphorous	Silicon
A	Fe410WA	0.23	1.5	0.05	0.05	0.4
B	Fe410WB	0.22	1.5	0.045	0.04	0.4
C	Fe410WC	0.20	1.5	0.04	0.04	0.4

1.7 Stress-Strain curve of mild steel



Stress-strain diagram for mild steel.

Stress-strain curve gives information to understand the behavior of material subjected to given loading condition.

OABCDEFG is a stress-strain curve of mild steel subjected to gradually increasing tensile load.

OA → represents a straight line, where stress is directly proportion to strain and it obeys Hooke's law.

A → represents limit of proportionality, stress beyond which linear variation ceases.

B → represents elastic limit, maximum stress up to which the material regains its original shape after removal of load.

AB → is not a straight line.

Stress beyond point B, material goes to plastic region and stress reaches to upper yield point C. At this stage material deforms and there will be reduction in cross-sectional area. Then stress reduces to a point D is known as lower yield point.

Between DE → when load remains constant, stress also remains constant. But there will be a considerable amount deformation.

From point E → strain hardening takes place, where the material starts to deform when it reaches yield point and becomes stronger. So addition stress is required to cause addition deformation of the material till it reaches to point F. This point is known as ultimate point and stress at this point is called ultimate strength.

At point F → necking takes place, where there will be a reduction of cross-section area of material at a rapid rate. Apparent stress reduces but true stress goes on increases until material breaks at point G and this pint is known as point of failure or fracture.

Failure of ductile material is cup and cone type and ultimate strength is calculated at 0.2% of maximum strain.

1.8 Structural steel products

- Flat hot rolled structural steel sections – plates, flat bars, sheets, strips etc.
- Hot rolled structural steel sections – hollow sections, rolled sections
- Cold rolled structural sections
- Bolts
- Welding electrodes

Rolled structural steel

They are two types 1) **Hot rolled structural steel** and 2) **Cold rolled structural steel.**

Hot rolled structural steel is a mill process in which rolling of steel at high temperature above recrystallization state where steel can be formed into shapes easily. This method is very cheaper and faster because there will be no delay in the process.

Cold rolled structural steel is a milling process in which steel is rolled at room temperature or below recrystallization state. This method increases the strength by 20% and having good surface finish with lesser dimensional tolerance.

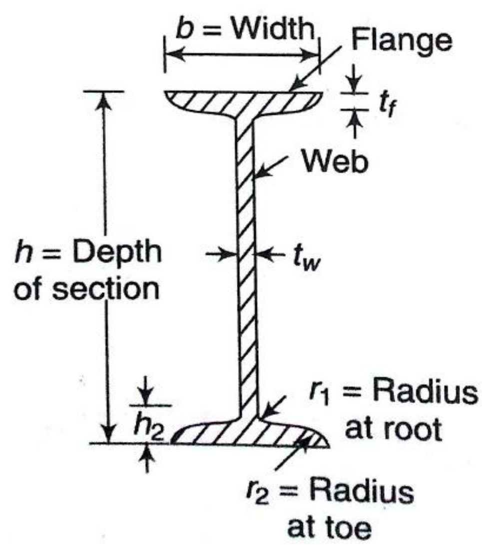
Hot rolled structural sections

- **I section:-**

These are used as columns and beams.

This section resists BM and SF; where 80% of BM will be resisted by flange and 20% will be resisted by columns. 95% of SF will be resisted by web and remaining will be resisted by flange.

Sometimes I sections with cover plates are used in order to resist large bending moments.



I sections are classified as

1) Indian Standard Joist/Junior Beams (ISJB)
2) Indian Standard Light Beams (ISLB)
3) Indian Standard Medium Beams (ISMB)
4) Indian Standard Wide Flange Beams (ISWB)

5) Indian Standard Heavy Beams (ISHB)

For example; ISMB 450@710.2N/m means ISMB with depth 450mm and having weight 710.2N per meter length.

ISMB, ISLB, ISJB are used as beams and ISWB & ISHB are employed as columns.

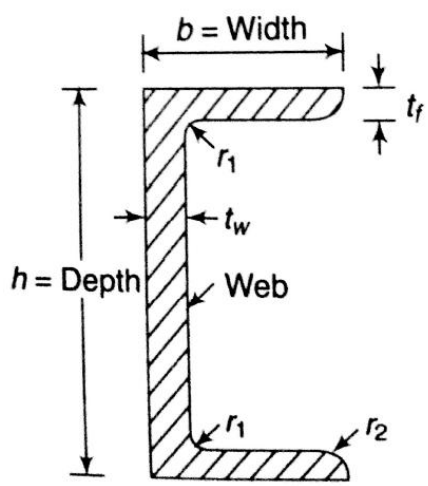
- **Channel section**:-

These are used as beams and columns.

Because of its shape, it allows to join angle sections to its web.

Built-up channels are conveniently used as columns.

Double channel members are used as bridge truss members.



Standard channel section are classified as

1) Indian Standard Junior Channels (ISJC)
2) Indian Standard Light Channels (ISLC)
3) Indian Standard Medium weight with sloping flange (ISMC)
4) Indian Standard Medium weight with Parallel flange (IMCP)

For example; ISMC 200@216.9N/m means ISMC with height 200mm and weight 216.9N per meter length.

- **Angle sections:-**

Abbreviated as ISA followed by widths and thickness.

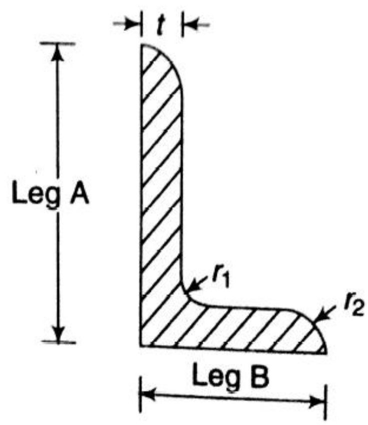
Available in equal and unequal sizes

Angles are used as connecting members for connecting structural elements.

Most employed in fabrication works.

Used as combination with plates to form plate girders.

Employed as tension and compression members of roof trusses.



Indian standard Angle sections are classified as

1) Indian Standard Equal Angles
2) Indian Standard Unequal Angles

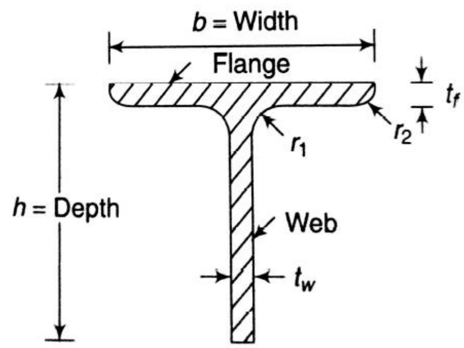
For example; ISA65X65X8 indicates an equal angle with legs of equal size 65mm and thickness 8mm.

ISA65X50X8 indicates an unequal angle with unequal leg widths with a thickness of 8mm.

- **T section:-**

Greater applications in furniture fabrication.

This type of section resists Bi-axial loadings.

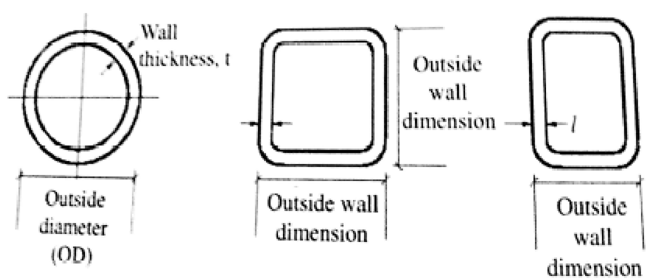


Indian standard T sections are classified as

1) Indian Standard Normal T bars (ISNT)
2) Indian Standard Long Legged T bars (ISST)
3) Indian standard Deep Legged T bars (ISDT)
4) Indian Standard Junior T bars (ISJT)
5) Indian Standard Light T bars (ISLT)
6) Indian Standard Wide T bars (ISWT)

- **Steel tubes:-**

These are used as scaffoldings, trusses, domes and columns.



The Structural Sections

- **Rolled steel flats:-**

Used for lacings in columns.

Used as ties.

Abbreviated with width followed by a letter F and thickness

For example; 50F8 where width ≤ 250mm.

- **Plates**:-

Designated using symbol PL followed by length, width and thickness.

For example; PL200X1000X6mm where T ≥ 5mm.

- **Steel sheets and strips:-**

Sheets are designated using symbol SH followed by length, width and thickness. T > 5mm

Strips are designated using symbol ST followed by width and thickness. T < 5mm

For example; SH2000X600X8 and ST200X3.

- **Square bars:-**

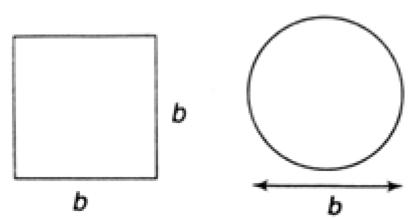
Steel bars of square section are made in various sizes from 5mm to 20mm and designated as ISSQ.

- **Round bars:-**

Round bars are used as reinforcements in RCC.

Solid circular bars are available in various from 5mm to 200mm.

Designated as ISRO.

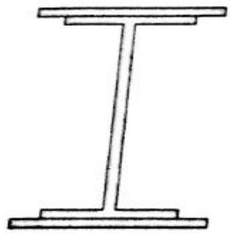


- **Built-up sections:-**

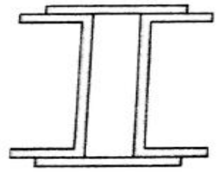
Rolled steel sections can be strengthened by welding cover plates on it.

Two rolled steel sections can be combined to resist loads in different directions

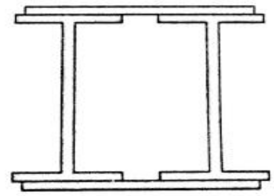
Two steel sections can be combined with lacing plates to act as single unit.



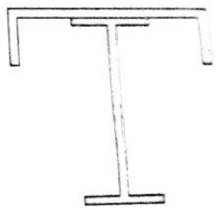
I-Section With Cover Plates



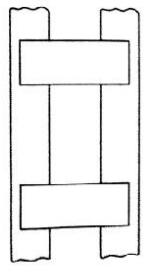
Double Channel Section With Cover Plates



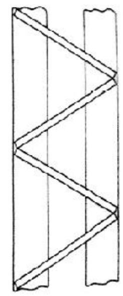
Double I-Section With Cover Plates



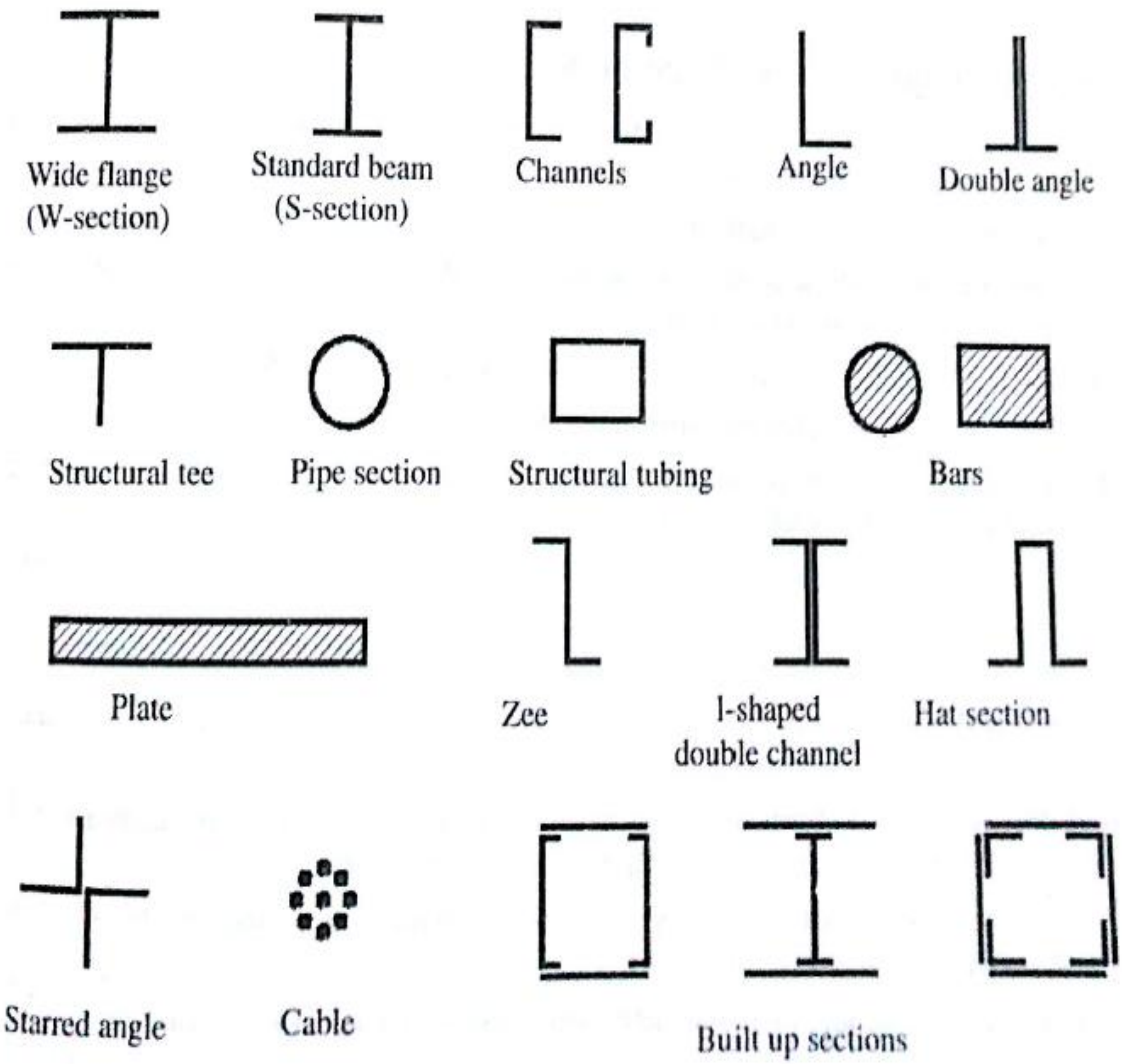
Crane Girder



Battened Member



Laced Member



Common structural shapes

1.9 Classification of cross-section

1) **Class I or Plastic section**:- This type of sections are subjected to bending moments equal to plastic moments and can undergo local rotation means plastic hinge is formed.

Plastic hinge refers to the deformation of part of beam wherever plastic bending takes place.

Plastic moment means moment at which entire cross section reaches to yield stress.

2) **Class II or Compact section**:- This type of sections are subjected to bending moments equal to plastic moment but cannot undergo local rotation.

3) **Class III or Semi-compact sections**:- This type of sections are subjected to bending moments equal to yield moments where cross section starts to buckle after outermost fiber yields.

   Yield moment is the moment at which only exterior fibers reaches yield stress.

4) **Class IV or Slender sections**:- This type of section fails before yield moment is obtained.

1.10 Theories of failure / Failure criteria of steel sections

1) **Maximum stress theory (Rankin's theory):-** It states that, yield occurs when maximum principal stress reaches to uni-axial tensile stress.

2) **Maximum strain theory (St. Venant's theory):-** It states that, yield occurs when maximum principal tensile strain reaches uni-axial tensile strain at yield.

3) **Maximum shear stress theory (Guest's and Trescas's theory):-** It states that, yield occurs when shear stress reaches half of uni-axial tensile stress at yield.

4) **Distortion or shear strain energy theory (Von Mises theory):-** It states that yield occurs when shear strain energy reaches shear strain energy in simple tension.

1.11 Types of steel structures

Buildings:- These may include rigid, semi-rigid or simple connected frames, load-bearing walls, cable-stayed and cantilevered structures. Buildings may be single or multi-storeyed, single bay or multiple bays. Buildings are also classified according to use, such as residential, commercial, office, industrial etc. these buildings may include a steel frames or a steel roof supported by load-bearing walls.

Bridges:- Bridges may be classified as truss, plate-girder, arch, cantilever, cable-stayed or cable suspension. The truss and plate-girder bridges are commonly adopted for small to moderate spans and cable-stayed and suspension bridges for long spans. Bridges may be classified as railway, road, road-rail and pedestrian bridges depending upon use.

Towers:- Towers may be of different types, such as lighting towers, power transmission towers, observation towers, towers for radar and TV installation, telephone rely towers and windmill towers. Towers may be self-supporting or cable-stayed. Most towers are made of steel angles or tubes, which are bolted at site.

Storage tanks:- These may be rectangular, circular, or spherical. They can be used to store water or oil. They may rest on ground or be elevated on a staging.

Other structures:- Silos, bunkers, domes, folded plates, offshore platforms, chimneys, cooling towers.

1.12 Advantages of steel structures

- Steel members have high strength per unit weight. Therefore steel members of small section having less self-weight can able to carry heavy loads.
- It has assured quality and durability.
- Being light, steel members can be conveniently handled and transported. For this reason pre-fabricated products are frequently provided.
- Properly maintained steel structures give longer life. The properties of steel do not change with time. This makes the steel as most suitable material for a structure.
- Steel is a ductile material which does not fail suddenly, but gives visible signs of impending failure by large deflections.
- Speed of construction is the other advantage of steel structure. When pre-fabricated products are ready at the site can be erected easily. This saves the construction time.
- Additions and alterations can be done easily to steel structures.
- Steel has highest scrap value amongst all building material.
- If joints are taken care, it is the best water and gas resistant which can be used to store water.
- Material is reusable.

1.13 Disadvantages of steel structures

- They are costly.
- It is susceptible to corrosion.
- They require regular maintenance. So maintenance cost is high.
- They become more elastic when exposed to fire.

- Needs high degree of control to ensure proper fitting of various structural elements.

1.14 Applications of steel structures

1) **Construction:-** Because high strength to weight ratio, slender sections can carry heavy load and can be obtained at affordable price. So long span structures can be constructed at low cost.

   Eg:- Many historical buildings like Empire State building primarily consists of steel as construction material. Steel also used in

   - RCC
   - Auditoriums
   - Low and high rise buildings
   - Piers and suspension cables
   - Educational institutes and hospitals.

2) **Transportation:-** Used in Engineering and manufacturing sectors but bulk goes to transport vehicles. Different types of steel are used in car body, doors, engines, gear box, steering, wheel axels etc. Other than automotive market, steel is used to transport materials by trains, rails, ships, tracks and transmission lines.

3) **Energy:-** Many sectors of power includes nuclear, natural gas, wind, electric needs steel for infrastructure. Many energy projects rely on steel for

   - Oil and gas wells and platforms
   - Electric transmission lines
   - Pipe lines
   - Transmission towers
   - Wind turbines

4) **Packages:-** Used to protect from water, light and air exposure and fully recyclable. Used for packing food, beverages followed by general lines and closures. (Eg:- bottle caps)

5) **Appliances:-** 75% of total weight of home appliances is from steel. Used in mixers, ovens, fridge, washing machine farm vehicles and machine tools etc.

1.15 Loads and load combinations

Loads:- For the purpose of safe designing a structure, it is essential to know what are the forces that are likely to act and the worst combination for which the structure should withstand during its life span.

1) **Dead load:-** These loads are nothing but self weight of the structure and this load is fixed in magnitude and in position. The load should be assumed as specified in IS 875(Part I) and checked after design is completed.

2) **Imposed load:-** These are also called as gravity loads other than dead load produced by occupancy of the structure. Imposed loads are different for different occupancy and should be selected as per IS 875(Part II). Crane load should be selected as per manufactures data. Snow load should be selected as per IS 875(Part IV). Different types of imposed are as follows

   a) Live load
   b) Crane load
   c) Snow load
   d) Dust load
   e) Wave load
   f) Earth pressure

   Live loads:- these are also known as imposed loads which vary in magnitude or in position. Furniture loads, moving equipments, weight of persons in assembly halls/school, parts stored in ware house etc., are some of the examples for live loads.

3) **Wind loads:-** For the purpose of safe design of structure, wind loads should be adopted from the loops and tables gives in IS 875(Part III).

4) **Seismic loads:-** If the structure is situated in earthquake prone areas, seismic force should to considered for the safe design of structures. Earthquake force causes ground motion in both directions (ie. Horizontal and Vertical), which can create enormous damage to the buildings. As per IS 1893:2002, loads should be selected for the safe design of structures.

5) **Accidental loads:-** Accidentals loads like fire, explosion, impact or collusion should be selected as per IS 875(Part V).

6) **Errection loads:-** These are temporary loads, that are likely to act at the time of construction.

Load combination:-

Following are the load combinations considered for the safe design of structures.

DL	DL + IL + EL
DL + IL	DL + IL + TL
DL + WL	DL + WL + TL
DL + EL	DL + EL + TL
DL + TL	DL + IL + WL + TL
DL + IL + WL	DL + IL + EL + TL

DL – Dead load, IL – Impose d load, WL – Wind load, EL – Earthquake load, TL – Temperature load, SL – Snow load.

When snow load is deposited on roofs, replace IL by SL.

Structural Analysis:- It is necessary to find the internal forces developed in the members of structure. The required internal forces for design are 1) Axial force and 2) Moments.

Different types of structural analysis are as follows

1) **Elastic Analysis:-** No fiber of the members have yielded for the design load and stress is directly proportional strain.

I order analysis:- It is based on load acting on un-deformed shape of the structure.

II order analysis:- It is based on deformed shape of the structure.

2) **Plastic analysis:-** It is assumed that when every fiber of the section reaches the yield stress, a plastic hinge is formed. Member can freely rotate without resisting any additional moment.

3) **Advanced analysis:-** For frame which is fully laterally restrained, an advanced analysis is carried out.

4) **Dynamic analysis:-** Carried out in accordance with IS 1893 (Part I).

1.16 Design philosophy

The aim of the design is to decide the shape, size and connection details of the members and structure being designed should perform satisfactorily during its life span. Therefore, structure should

- Sustain all loads that are likely to act on it
- Sustain deformation during and after construction
- Adequate durability
- Resistance to fire
- Be stable and have alternate load path such that building should not collapse due to accidental loadings.

Design philosophies are listed below

1) Working Stress Method (WSM)
2) Ultimate Load Method (ULM)
3) Limit State Method (LSM)

1) **Working stress method:-** This is the oldest, systematic and analytical design method. In this method stress is directly proportional to strain up to yield stress. Therefore, the stress caused due to loads should be checked against allowable or permissible stress which is a fraction of yield stress.

Permissible stress = $\frac{\text{Yield stress}}{\text{Factor of safety}}$

In working stress method, working stress ≤ permissible stress

Advantages of working stress method

- Method is simple
- As working stress are low, serviceability requirements are satisfied.

Disadvantages of working stress method

- This method provides un-economical sections.
- This method gives an idea that, factor of safety multiplied by working load gives failure load. But this not true.

2) **Ultimate load method:-** the limitation of working stress method in accessing the actual load carrying capacity leads to the development of ultimate load method or **load factor method.** This method is based on plastic analysis, means even when the fiber of the section reaches the yield, the member can resist load till sufficient plastic hinge is formed to develop a collapse mechanism.

Load factor = $\frac{\text{Design load}}{\text{Working load}}$

Advantages of load factor method

- It allows the selection of load factor based on load combination.
- The redistribution of internal forces are accounted.

Disadvantages of load factor method

- No guarantee for serviceability of structure.

3) **Limits State Method:-** This method take care of both strength and serviceability requirements. Limit state is the condition in which structure is considered to be failed to serve the purpose for which it is intended.

- **Limit state of strength:-** Refers to the loss of equilibrium or stability of structure. It deals with strength, overturning, sliding, buckling, fatigue and brittle fracture.
- **Limit state of serviceability:**- It is an acceptable limit of performance of structure. It deals with discomfort to occupancy or malfunction caused by excessive deflection, vibration, cracking, corrosion, durability and fire.

Characteristic strength:- It is the strength of the material below which not more than prescribed percentage of test results are expected to fall (allowable % is 95%)

Characteristic yield strength:- It is the stress below which not more than 5% of test results are expected to fall.

Characteristic load:- It is the value of load not being exceed during the life span of structure. (Loads on the structure does not exceed 95% times the characteristic loads)

Partial safety factor is regarded as overloading factor for which structure is designed. Partial safety factor is different for different material and loads.

Design actions/loads (Q_d) = $\sum_k \gamma_{fk} Q_{ck}$

Design strength S_d = $\frac{S_u}{\gamma_m}$

1.17 **Design considerations:-** Aim of the structural designer is to design the structure such that it should perform satisfactorily during its life time.

- **Safety:-** This is one of the factor for any structure. The structure may collapse not only due to expected loads and also due to accidental or impact loads and other possibilities such as expose to load exceeding load bearing capacity, overturning, sliding, buckling, fatigue and brittle fracture. Another aspect of safety is stability means stable in its equilibrium configuration.

- **Serviceability:-** Directly related to the performance of the structure under service loads should not cause discomfort for the user due to excessive deformation, vibration, crackling, fire, durability and corrosion. In some cases design may satisfy the safety criteria but not satisfies the serviceability criteria.

- **Economy:-** By increasing the design standards, cost of the structure also increases. For overall economy, one should look after the initial cost and maintenance cost.

- **Environment friendly:-** While selecting a material, one should look after the long-term environmental effects such as dismantling, maintenance, recyclability, repair and retrofitting.

- **Aesthetics:-** External appearance should be good after the construction of structure.

1.18 Codes and specifications

A structural engineer is often guided in his efforts by the code of practice. A code represents the varieties of opinions of engineers and professionals. It may not cover in detail every situation a designer may encounter. Often the designer must exercise his own judgment in interpreting and applying the requirements of code. It has to be noted that, by strictly following the codes may often give number of innovative designs.

Codes are basically written for the purpose of protecting public. They provide guidelines for the design and construction of structures. They are revised at regular intervals to reflect new developments in research, materials, construction techniques, experience gained from past design practice, behavior of existing structures and failure of structures.

Codes contain recommended loads for a given locality and recommended fire and corrosion protection. They also contain rules governing the ways in which loads are to be applied and design rules for concrete, steel and other materials.

The code should serve the following functions

- They ensure adequate structural safety, by specifying minimum requirements for the design.

- They ensure consistency among different engineers.

- They protect engineers from disputes, though codes may not provide legal protection.

In India, the Bureau of Indian Standards issues the codes and standard handbooks. Some of them are listed below

a) **IS 800: 2007** – Code of practice for general construction in steel.

b) **IS 875: 1987** – Code of practice for design loads for buildings and structures.

IS 875:1987 Part I – Dead loads

IS 875:1987 Part II – Imposed loads

IS 875:1987 Part III – Wind loads

IS 875:1987 Part IV – Snow loads

IS 875:1987 Part V – Special loads and load combinations.

c) **IS 1893: 2002** – Criteria for earthquake-resistant design of structures.

d) **IS 808: 1989** – Dimensions for hot-rolled steel beams, columns, channels and angle sections.

1.19 Structural Elements

A steel frame building consists of a skeleton frame which supports all the loads to which the building is subjected. The various elements of steel framed buildings are as follows.

1) **Beams and girders:-** These support vertical loads and are therefore subjected to bending moments and shear forces.

2) **Ties:**- These are the members subjected to axial tension.

3) **Struts, columns and stanchions:**- These are the members subjected to compressive loads. They may in addition be subjected to bending moments.

4) **Truss and lattice girders:**- These are framed members consisting of compression and tension members. These framed units carry lateral loads.

5) **Purlins:-** These are the members meant to support roof sheeting.

6) **Bracings:-** Theses are diagonal ties and struts connected to columns and roof trusses. They are meant to support wind loads. They provide stability to the building.

7) **Joints:**- Joints are provided to connect members.

8) **Bases:**- Bases are meant to transmit loads from column to foundation.

1.19 Assignment questions

1) What are the advantages and disadvantages of steel structures?
2) Explain the failure criteria's of steel structures.
3) Write a note on load and load combinations.
4) Explain the classification of cross section of elements?
5) What are the applications of steel structures?

1.20 Outcome

- Gives an idea about the stress strain behavior of mild steel
- Gives an idea about different structural steel products
- Gives an brief note on advantages and disadvantages of steel structures over other constructions materials.

1.21 Future study

- One can study the design methodology of each structural steel products which are necessary to build a structure.

(Unit - 4) Plastic Behavior of Structural Steel

CONTENTS

Introduction

The traditional analysis of structures is based on the linear elastic behavior of materials, implying that the material follows Hooke's law (Stress is proportional to strain). It is also assumed that the deformations are small, implying that the original dimensions of the structure can be used in the analysis. This is also known as first order elastic analysis. (Cl. 4.4.2 pp - 24)

IS 800 - 2007 permits plastic analysis as per the Cl. 4.5 (pp 25 and 26). However, the requirements specified in Cl. 4.5.2 shall be satisfied unless otherwise specified.

- The yield stress of the grade of structural steel used shall not exceed 450 MPa.
- The stress - strain characteristics of steel shall comply with IS: 2062 to ensure complete plastic moment redistribution.
- The stress - strain diagram shall have a plateau at the yield stress level extending for at least six times the yield strain.
- The ratio of ultimate tensile stress to the yield stress for the specified grade of steel shall not be less than 1.2
- The percentage elongation shall not be less than 15 and the steel shall exhibit strain - hardening capabilities. (Steel confirming to IS: 2062 shall be deemed to satisfy the above requirements)
- The members shall be hot - rolled or fabricated using hot - rolled plates and sections.
- The cross section of the members shall be plastic (class 1 section) at plastic hinges and elsewhere at least compact sections. (class 2 section) Table 2 shall be followed in this regard.
- The cross section shall be symmetrical about the axis perpendicular to the axis of the plastic hinge rotation indicating that the beams shall be symmetrical about y-y axis and columns shall be symmetrical about both y-y and z-z axes.
- The members shall not be subjected to impact and fluctuating loading requiring fracture and fatigue assessment.

Objective

1. To understand the plastic theory
2. To understand the concept of plastic hinge concept
3. To understand the concept of plastic moment
4. To understand the concept of plastic collapse load

Stress - strain curves of structural steel

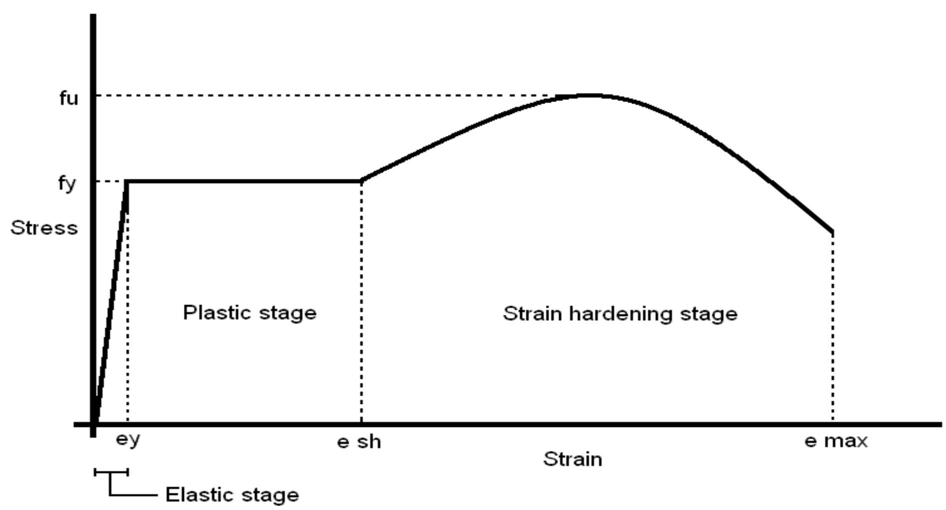
A typical stress - strain curve of steel confirming to IS: 2062 is shown in the figure. where,

f_y = yield stress in MPa e_y = yield strain

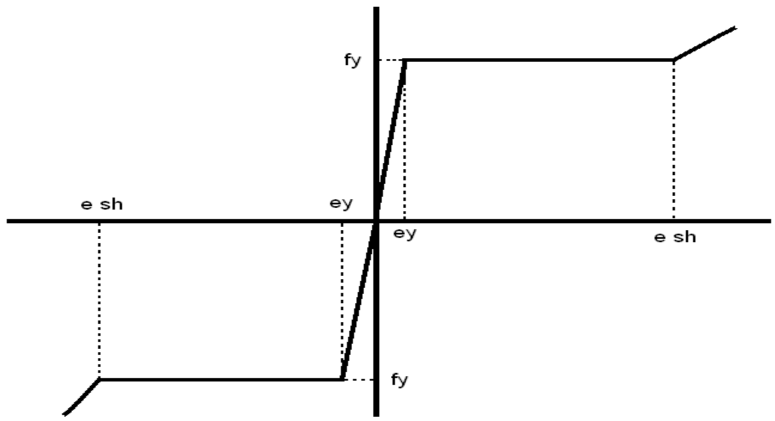
f_u = Ultimate stress in MPa e_{sh} = strain hardening strain e_{max} = ultimate strain

$e_{sh} = 6 * e_y$, $e_{max} = 180 * e_y$ and $f_u = 1.2\ f_y$ (Typical)

From the stress - strain curve, steel yields considerably at a constant stress due to large flow of the material. This property known as ductility enables steel to undergo large deformations beyond the elastic limit without danger of fracture. This unique property of steel is utilized in plastic analysis of structures.



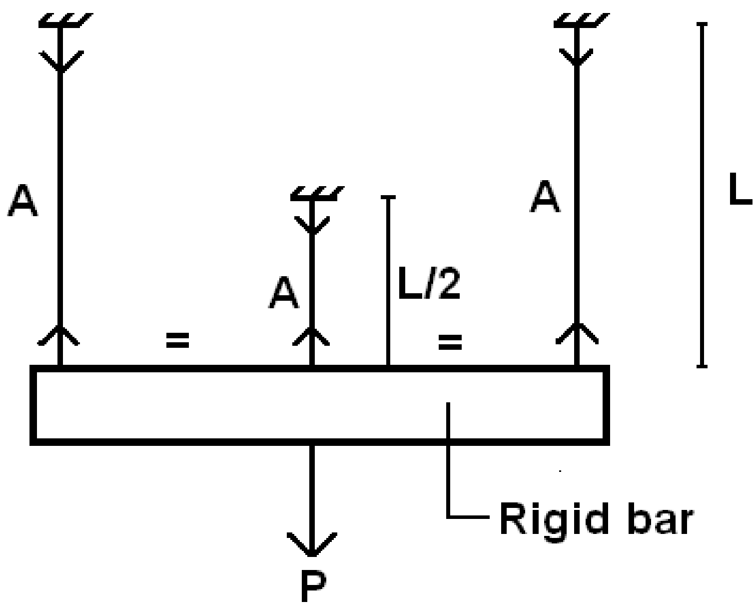
Stress - Strain Curve (Typical)



Perfectly Elasto - Plastic Material (Typical)

Calculation of failure loads in simple systems

Consider a three bar system shown below of length and area of C/S of each bar as indicated. E is the modulus of elasticity of the material.



Elastic analysis (Strength of Materials approach):

P_1 is the force in outer bars

P_2 is the force in the middle bar

Using ΣV = 0, (Vertical equilibrium equation)

$2P_1 + P_2 = P$ (1)

By compatibility, elongation of each bar is same -

$P_1L/AE = P_2L/2AE$

from which

$P_1 = P_2/2$ or $P_2 = 2P_1$ (2) Substituting (2) in (1),

$2P_1 + 2P_1 = P$ or $P_1 = P/4$ and $P_2 = P/2$

In elastic analysis, as $P_2 > P_1$ the middle bar reaches the yield stress first and the system is assumed to fail.

$P_2 = f_yA$ and $P_1 = f_yA/2$

Yield load = $2P_1 + P_2 = 2f_yA$ ----- Maximum load by elastic analysis

Plastic analysis:

In plastic analysis, it will be assumed that even though the middle bar reaches the yield stress, they start yielding until the outer bars also reaches the yield stress. (Ductility of steel and redistribution of forces) With this, all the bars would have reached yield stress and the failure load (or ultimate load or collapse load) is given by

Collapse load, $P_u = 2\ f_y\ A + f_y\ A = 3f_yA$ ------ Maximum load by plastic analysis

The collapse load calculated by plastic analysis is 1.5 times that of the elastic analysis. (Reserve strength) Plastic analysis can give economical solutions.

Plastic Theory of Beams:

The simple plastic theory makes use of the ductility of steel. (Large strain at collapse). The following assumptions are made in plastic bending of beams -

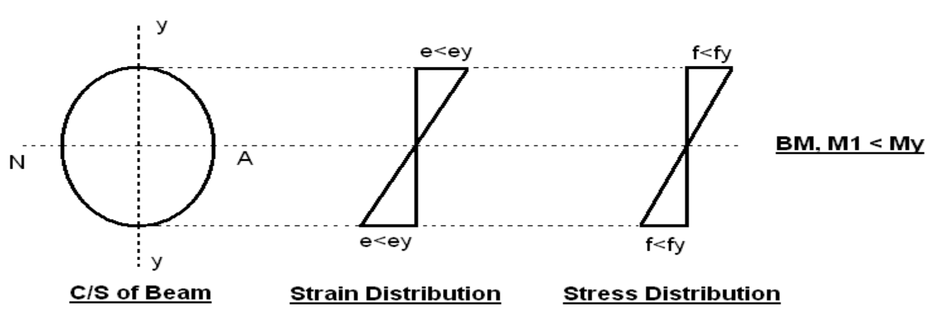
- Structural steel is a ductile material capable of deforming plastically without fracture.
- The material is homogeneous and isotropic obeying Hooke's law up to limit of proportionality (yield point) and then the stress is constant with increase in strain.
- The stress - strain curve can be represented by an ideal elasto - plastic material with properties of steel in compression and tension same. (yield stress and yield strain, modulus of elasticity etc.,)

- Cross - sections remain plane and normal to the longitudinal axis before and after bending. With this, the effect of shear force is ignored and the distribution of the strain across the depth of the c/s of the beam is linear.
- The effect of axial forces and residual stresses are ignored.
- The c/s of the beam is symmetrical about an axis parallel to the plane of bending. (y -y axis)
- Members are initially straight and instability does not develop before collapse occurs due to the formation of sufficient plastic hinges.
- Each layer of the beam is free to expand or contract independently with respect to the layer above or below it (each layer is separated from one another)
- Deformations are sufficiently small so that $\theta = \tan\theta$ can be used in the calculations of the collapse load.
- The connections provide full continuity so that plastic moment can develop and transmitted through the connections.
- Strain energy due to elastic bending is ignored.

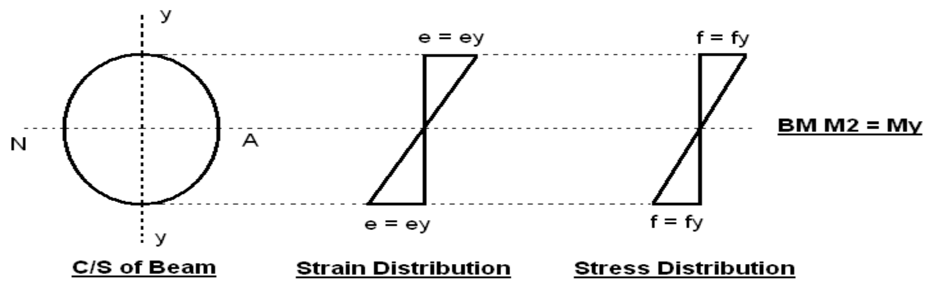
Behavior of beam under an increasing BM:

Consider a beam having a symmetrical C/S subjected to an increasing BM.

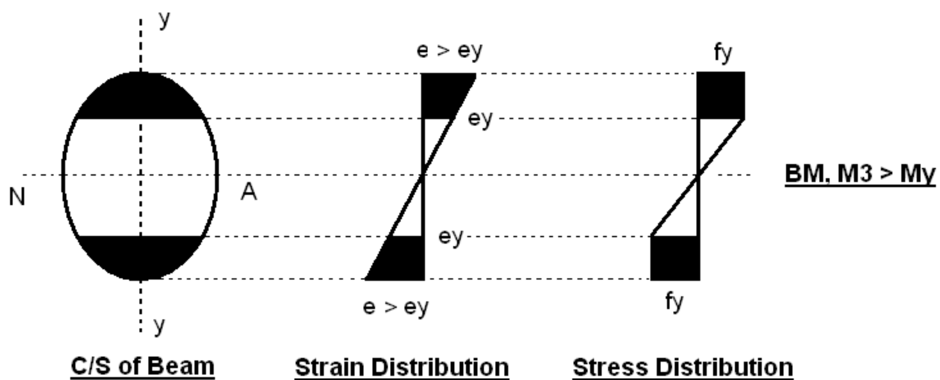
With BM $M_1 < M_y$ (yield moment) the stress and strain distributions across the depth will follow the elastic bending equation (Euler's - Bernoulli's equation) and is indicated in the figure below. All the fibres are stressed below the yield stress, f_y.



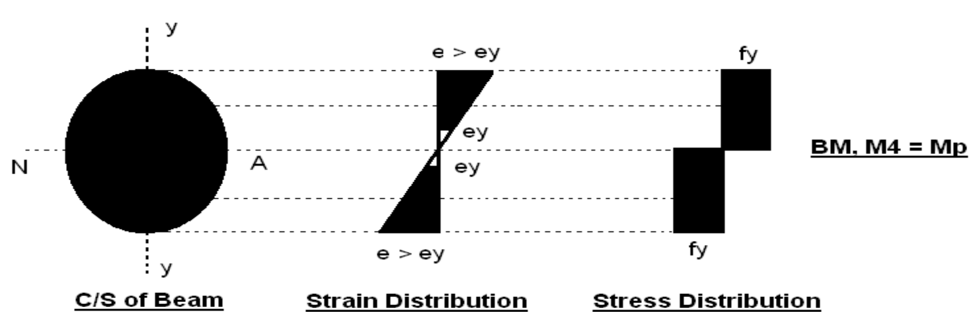
With BM, $M_2 = M_y$, the extreme fibres of the c/s begin to yield and the stress will be equal to f_y .The stress and strain distributions across the depth will still follow the elastic bending equation (Euler's - Bernoulli's equation) and is indicated in the figure below. All the fibres other than extreme fibres are stressed below the yield stress, f_y. The permissible stress (working stress) method is based on this behavior with suitable factor of safety.



With further increase in BM, $M_3 > M_y$, yielding spreads into inner fibres. The c/s is elasto - plastic and the internal moment of resistance can be computed, if required from the stress distribution shown below.



Finally, the yielding of fibres spreads almost for the entire c/s and the BM, M_4 at this stage is called the plastic moment M_P .The c/s is said to be fully plasticized. A plastic hinge is formed at this stage. After this, the deflection increases rapidly resulting in collapse or failure. The neutral axis, NA is now called the Equal Area Axis. An idealized stress distribution is shown below.



Basic Definitions: Plastic Moment, Mp:

It is defined as the moment of resistance of a fully plasticized or yielded c/s. The entire c/s is under yield stress, f_y.

Mathematically, $M_P = f_Y Z_{pz}$

f_y is the yield stress of the material

Z_{pz} is the plastic section modulus of the c/s about z-z axis

Yield Moment, My:

It is defined as the moment of resistance of a c/s whose extreme fibres only has reached yield stress, f_y.

Mathematically, $M_y = f_Y Z_{ez}$

f_y is the yield stress of the material

Z_{ez} is the elastic section modulus of the c/s about z-z axis

Plastic hinge:

It is defined a point in a flexural member where full plastic moment has developed and is rotating at a constant moment, M_P. The following characteristics are observed with respect to a plastic hinge :

- A plastic hinge is a zone of yielding in a flexural member, where the stress is equal to f_Y throughout the c/s.
- The length of the plastic hinge depends on the type of loading, c/s and support conditions. However, in the analysis, it is assumed as point where all the plastic rotation takes place.
- At the point, where plastic hinge is formed, it will be rotating with a constant plastic moment, Mp
- Plastic hinges are formed at points where concentrated loads are acting, fixed or continuous supports in an indeterminate beam, joints in a rigid frame and at points of zero shear, when UDL or other varying loads are acting.
- Plastic hinge is formed in a member of lesser capacity, when two members meet as in the case of a continuous beam.
- The first plastic hinge is formed at the point of maximum BM.

- The formation of plastic hinges allow redistribution of moments until plastic hinges are formed at all the critical sections.
- The formation of sufficient number of plastic hinges convert the structure into a mechanism where the given structure breaks into rigid links with large deformations. (Collapse)

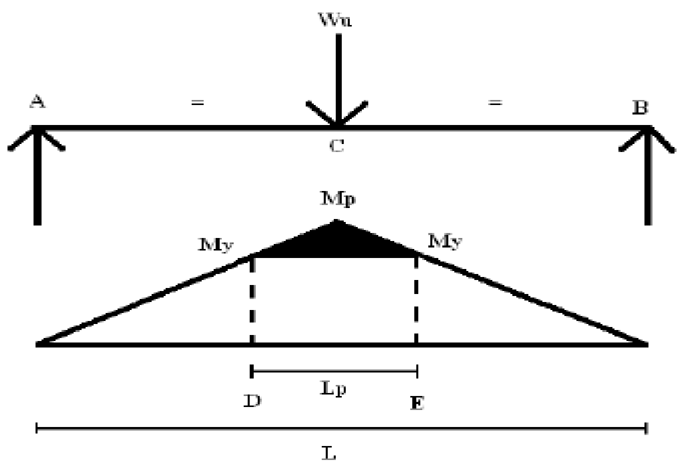
Length of plastic hinge:

It is the length in a beam over which the BM is greater than the yield moment, My

Problems

Example 1

Consider a simply supported beam subjected to a UDL which has yielded over the shaded area as shown. Let Lp and L be the length of the plastic hinge and span of the beam.



Consider a simply supported beam subjected to a central concentrated load which has yielded over the shaded area as shown. Let Lp and L be the length of the plastic hinge and span of the beam.

From similar triangles, $L_p / L = (M_p - M_y) / M_p$

$= (1 - M_y / M_p)$

$= [1 - 1/ (M_p / M_y)]$

$= (1 - 1/S)$

Where $S = (M_p / M_y)$ is called the shape factor

Hence, $L_p = (1 - 1/S) L$

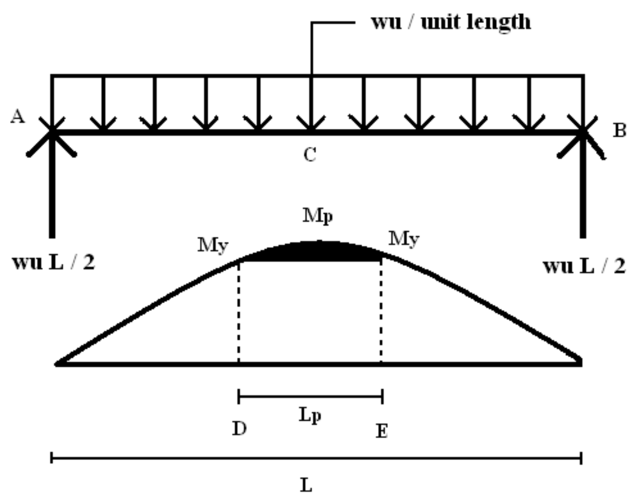
For a rectangular section, S = 1.5 and L_p = 1/3 * L

For typical I - sections, S = 1.14 and L_p = 0.123 L, say 1/8 * L

Length of plastic hinge depends on the shape factor of the c/s

Example 2

Consider a simply supported beam subjected to a UDL which has yielded over the shaded area as shown. Let Lp and L be the length of the plastic hinge and span of the beam.



Let $x = L_p / 2$

$M_y = w_u L / 2 * (L / 2 - x) - w_u (L / 2 - x)^2 / 2$

$= w_u L^2 / 4 - (w_u L / 2) * x - w_u / 2 * (L^2 / 4 + x^2 - L x)$ Cancelling $(w_u L / 2) * x$, we have

$M_y = w_u L^2 / 8 - w_u x^2 / 2$

$= w_u L^2 / 8 - w_u L^2 / 8 * (4 x^2 / L^2)$

Noting $M_p = w_u L^2 / 8$, we have

$M_y = M_p (1 - 4 x^2 / L^2)$

$M_y / M_p = (1 - 4 x^2 / L^2)$

$1 / S = (1 - 4 x^2 / L^2)$

$4 x^2 / L^2 = 1 - 1 / S$

$x^2 = L^2 / 4 (1 - 1 / S)$

$x = L / 2 \sqrt{(1 - 1 / S)}$

$L_p / 2 = L / 2 \sqrt{(1 - 1 / S)}$ $L_p = L \sqrt{(1 - 1 / S)}$

For a rectangular section, S = 1.5 and $L_p = 0.577 * L$

For typical I - sections, S = 1.14 and $L_p = 0.35 * L$

Length of plastic hinge depends on the shape factor of the c/s

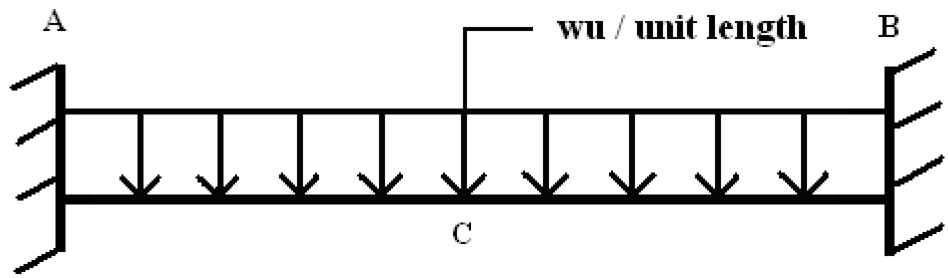
Note:-

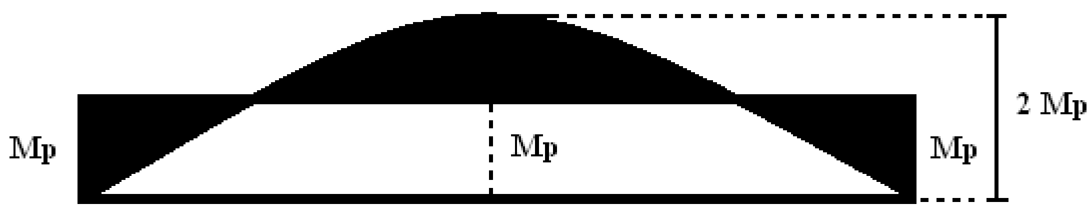
From the above examples, it is observed that the length of the plastic hinge is dependent on the loading and shape factor. It is also dependent on the support conditions. For propped cantilevers, fixed beams, continuous beams and rigid frames, it is still difficult to calculate the length of plastic hinges for different loadings and supports. Hence, it is customary to assume the plastic hinge in the analysis as a point where all the plastic rotation takes place.

Redistribution of Moments:

In determinate structural systems like simply supported beams, cantilevers and overhang beams, only one plastic hinge will be formed at the point of maximum BM and the structure is assumed to collapse. The increase in load carrying capacity is only marginal and depends on the shape factor. However, in indeterminate structural systems like propped cantilevers, fixed beams, continuous beams and rigid frames, there is further increase in load carrying capacity due to the redistribution of moments. This is called the reserve strength of the beam. This is mainly due to the property of ductility of steel.

Redistribution of moments is explained with respect to a fixed beam subjected to UDL throughout. The beam is shown in the figure below.





The maximum elastic BM occurs at the supports and is equal to wL^2 / 12. As a first step, the plastic hinges are formed at the supports and complete plasticization of the support c/s takes place. But this do not mean failure or collapse occurs. This is because a plastic hinge at the centre should also be formed. (No. of plastic hinges in indeterminate structures = SI + 1) At this stage, the plastic hinges formed at the supports will be rotating at constant moment, M_p due to ductility of the material. Collapse of the beam occurs only when complete plasticization occurs at the centre resulting greater load carrying capacity.

At collapse, using equilibrium method, we have

$w_u L^2 / 8 = 2 M_p$

$w_u = 16 M_p / L^2$ ----------- Collapse load for the beam

At first yield, $M_y = w_y L^2 / 12$ at the supports, (only extreme fibres will yield)

$w_y = 12 M_y / L^2$

Reserve strength = $w_u / w_y = (16 M_p / L^2) / (12 M_y / L^2)$

$= 4 / 3 * M_p / M_y$

$= 4 / 3 * S$

where S is the shape factor.

For typical I - sections, S = 1.14

Reserve strength = 4 / 3 * 1.14 = 1.52

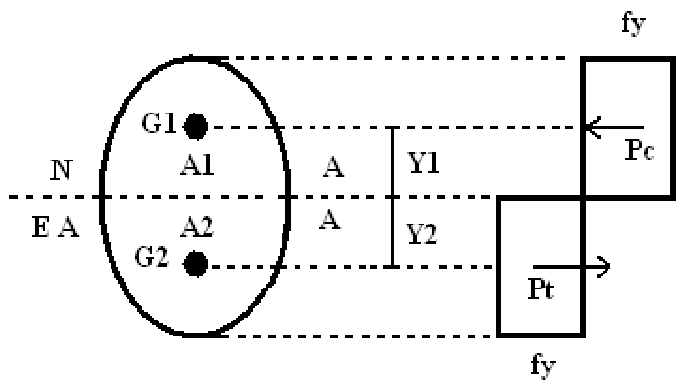
The beam is capable of carrying 52% more load beyond the first yield.

In general, increase in load carrying capacity occurs in plastic analysis due to - i) Plasticization of the section (formation of plastic hinges at critical sections)

ii) **Redistribution of moments**

Plastic Section modulus of a c/s:

Consider a c/s symmetrical about the vertical y-y axis as shown below with the section fully plasticized.



A_1 and A_2 are the areas above and below the Neutral Axis (Equal Area Axis)

f_Y is the yield stress in the material

Compressive force above the NA = $f_y A_1$

Tensile force below the NA = $f_y A_2$

For equilibrium, the forces are equal -

$f_y A_1 = f_y A_2$ from which

$A_1 = A_2$

For a fully plasticized c/s, area above the NA = area below the NA

In other words, $A_1 + A_2 = A$ $A_1 + A_1 = A$ $A_1 = A_2 = A/2$

The neutral axis (NA) of a fully plasticized section is called the equal area axis (EAA)

The plastic moment of a fully plasticized section can be obtained by taking moments of the forces above and below the EAA about EAA.

$M_p = f_y A_1 Y_1 + f_y A_2 Y_2$

$= f_y A/2\, Y_1 + f_y A/2\, Y_2$

$= f_y A/2\, (Y_1 + Y_2)$

$= f_y Z_{pz}$

where, Z_{pz} = plastic section modulus of the c/s about z-z axis = $[A/2\, (Y_1 + Y_2)]$

The plastic section modulus of a c/s, Z_{pz} is defined as the moment of the areas above and below EAA about the EAA. It is the resisting modulus of a completely plasticized section and is a geometric property.

Shape factor, S:

It is defined as the ratio of plastic moment, M_P to yield moment, M_Y. Also, called form factor.

Mathematically, $S = M_P / M_y$

$= f_Y Z_{pz} / f_Y Z_{ez}$

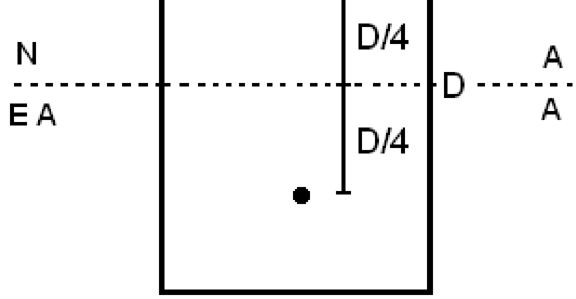
$= Z_{pz} / Z_{ez}$

It is also the ratio between the plastic section modulus to the elastic section modulus of a c/s and is a geometric property

Shape factor of simple sections:

1. **Rectangle:**

Consider a rectangle as shown



Elastic section modulus, Z_{ez} = I_{NA} / extreme fibre distance

= $(bD^3/12) / (D / 2)$

= $bD^2 / 6$

Plastic section modulus, Z_{pz} = Moments of the areas above and below the E A A (equal area axis) about the E A A.

$Z_P = [(bD / 2) \times D/4] \times 2$

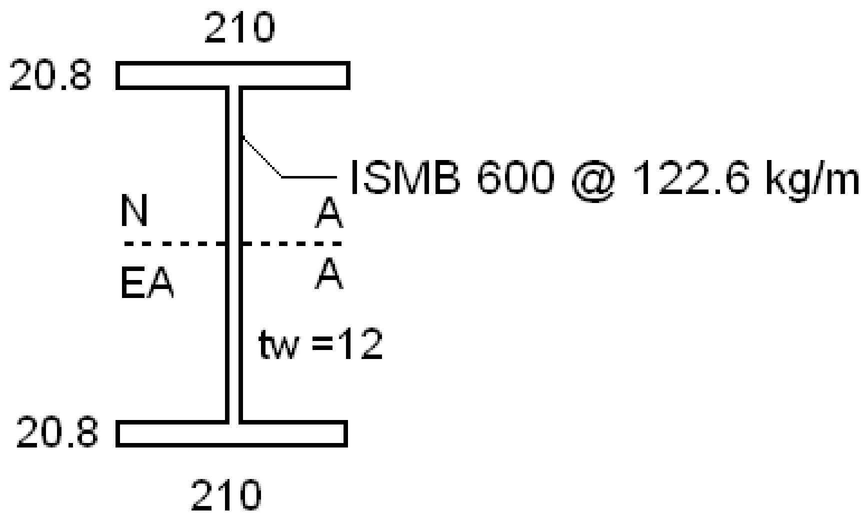
= $bD^2 / 4$

S = Z_{pz} / Z_{ez}

= 1.5

2. I - section, say ISMB 600 @ 122.6 kg/m:

Consider the I - section shown



z_{ez} = 3060.4 cm^3 (From hand book; Table I, pp - 4,5) = 3060.4 x 10^3 mm^3

z_{pz} = [210 x 20.8 x (600/2 - 20.8/2) + 12 x (600/2 - 20.8) x (600/2 - 20.8)/2] x 2

= 3465.38 x 10^3 mm^3 (The value given in Table 46; pp - 138 is 3510.63 x 10^3 mm^3)

Shape factor, S = Z_{pz} / Z_{ez}

= 1.132 (The value given in Table 46; pp - 138 is 1.147)

If the above c/s is used over an effective span of 8m, the permissible UDL on the beam is calculated as -

Yield Moment, M_Y = f_Y Z_{ez} (Assuming f_y =250 MPa)

= 250 x 3060.4 x 10^3

= 765.1 x 10^6 N mm

= 765.1 kN m

Plastic Moment, M_p = f_Y Z_{pz}

= 250 x 3465.38 x 10^3

= 866.35 x 10^6 N mm

= 866.35 kN m

Working bending moment, M = M_p / 1.5 = 577.57 kN m

If the permissible UDL on the beam is w/unit length, $w L^2 / 8 = M$

$w \times 8^2/8 = 577.57$

w = 72.2 kN/m (including self weight)

On similar lines, if central concentrated load is required, $W L / 4 + w_{self\ weight} * L^2 / 8 = M$

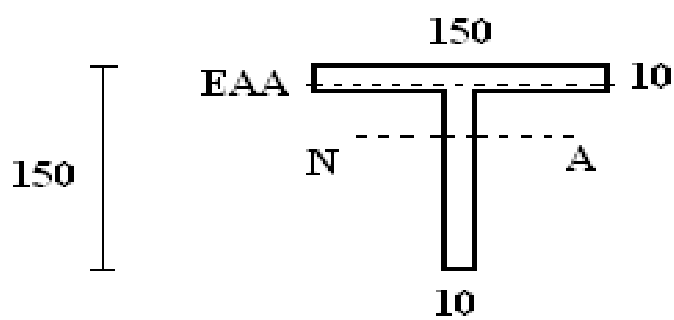
$W * 8 / 4 + 1.23 \times 8^2 / 8 = 577.57$

W = 283.87 kN

If the c/s is symmetrical about the horizontal z - z axis or NA, then both NA and EAA will co inside.

3. T - section say ISNT 150:

Consider the T section as shown



.

$Z_{ez} = 54.6\ cm^3$ (From hand book; Table VI pp - 18,19)

The section is unsymmetrical about the NA and hence the EAA has to be located. Total area of the c/s = 29.08 cm^2 = 2908 mm^2 (From hand book; Table VI, pp - 18,19) $A_1 = A_2 = A / 2 = 1454\ mm^2$

If EAA is at distance y from top, we have

150 * y = 1454 y = 9.69 mm

The plastic section modulus is calculated by taking moments of the area above and below the EAA about EAA.

Z_{pz} = 150 * 9.69 * 9.69 / 2 + 150 * 0.31 * 0.31 / 2 + 10 * 140 * (140 / 2 + 0.31)

= 105.48 x 10^3 mm^3

(0.31 = 10 - 9.69 ; 140 = 150 - 10) S = Z_{pz} / Z_{ez} = 1.93

Yield Moment, $M_Y = f_Y Z_{ez}$ (Assuming f_y =250 MPa)

= 250 x 54.6 x 10^3
= 13.65 x 10^6 N mm
= 13.65 kN m

Plastic Moment, $M_p = f_Y Z_{pz}$

= 250 x 105.48 x 10^3

= 26.37 x 10^6 N mm

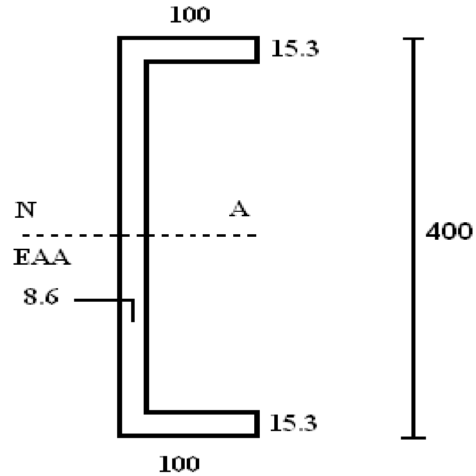
= 26.37 kN m

Working bending moment, M = M_p / 1.5 = 17.58 kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

4. Channel Section, say ISMC 400 @ 49.4 kg/m

Consider the channel section as shown.



The elastic properties of the section is obtained from Table II, pp 6 - 7 of ISI handbook [SP (6) - 1)]

Z_{ez} = 754.1 cm^3 = 754.1 x 10^3 mm^3

The beam is symmetrical about z - z axis and hence, NA and EAA co inside.

Z_{pz} = (100 * 15.3 * 192.35 + 8.6 * 184.7 * 184.7 / 2) * 2 = 881.97 x 10 mm^3

[192.35 = (400 / 2) - 15.3 / 2 ; 184.7 = (400 / 2) - 15.3]

S = Z_{pz} / Z_{ez} = 1.17

Yield Moment, $M_Y = f_Y Z_{ez}$ (Assuming f_y =250 MPa)

= 250 x 754.1 x 10^3

= 188.53 x 10^6 N mm

= 188.53 kN m

Plastic Moment, $M_p = f_Y Z_{pz}$

= 250 x 881.97 x 10^3

= 220.5 x 10^6 N mm

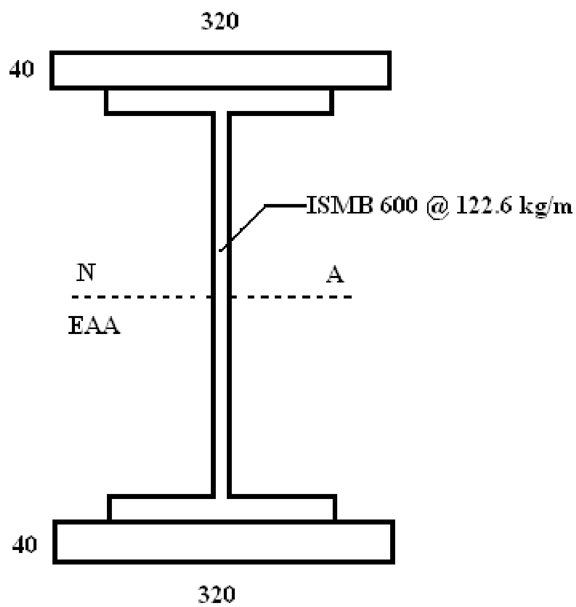
= 220.5 kN m

Working bending moment, M = M_p / 1.5 = 147 kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

5. Built up section, ISMB600 @ 122.6kg/m with 320 x 40 plates symmetrically placed

Consider the built up section as shown.



The elastic properties of the section is obtained from Table XIV, pp 54 - 55 of ISI handbook [SP (6) - 1)]

Z_{ez} = 10420.5 cm^3 = 10420.5 x 10^3 mm^3

The beam is symmetrical about z - z axis and hence, NA and EAA co inside

Z_{pz} = 3510.63 x 10^3 + (320 * 40 * 320) * 2

= 11702.63 x 10^3 mm^3

(The value of Z_{pz} of ISMB 600 @ 122.6kg/m is 3510.63 cm^3 obtained from Table 46 of IS : 800 - 2007 pp - 138; 320 = 600 / 2 + 40 / 2)

S = Z_{pz} / Z_{ez} = 1.123

Mean thickness of flange = (320 * 40 + 210 * 20.8) / 320 = 53.65 mm (Table XIV gives mean thickness of flange = 53.3 mm)

(210 and 20.8 = width of flange and thickness of flange of ISMB 600 @ 122.6kg/m) Mean thickness > 40 mm

f_y = 230 MPa (Table 1 - IS: 800 - 2007, pp - 14)

Yield Moment, $M_Y = f_Y Z_{ez}$ (Assuming f_y =250 MPa)

$= 230 \times 10420.5 \times 10^3$

$= 2396.72 \times 10^6$ N mm

= 2396.72 kN m

Plastic Moment, $M_p = f_Y Z_{pz}$

$= 230 \times 11702.63 \times 10^3$

$= 2691.6 \times 10^6$ N mm

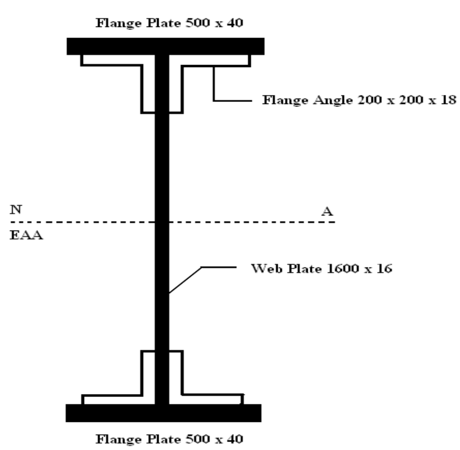
= 2691.6 kN m

Working bending moment, $M = M_p / 1.5 = 1794.4$ kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

6. Plate girder section, say Two 500 x 40 flange plates (one on each side), Four flange angles (two on each side) and One web plate 1600 x 16

Consider the section as shown.



The elastic properties of the section is obtained from Table XX, pp 84 - 85 of ISI handbook [SP (6) - 1)]

$Z_{ez} = 56782.9\ cm^3 = 56782.9 \times 10^3\ mm^3$

The beam is symmetrical about z - z axis and hence, NA and EAA co inside

Z_{pz} = (500 * 40 * 820 + 200 * 18 * 791 * 2 + 18 * 182 * 691 * 2 + 16 * 800 * 400) * 2

= 63485.26 x 10^3 mm^3

(820 = 1600 / 2 + 40 / 2 ; 791 = 1600 / 2 + 18 / 2 ; 182 = 200 - 18 ; 691 = 1600 / 2 - 18 - 182 / 2 ; 800 = 1600 / 2 ; 400 = 800 / 2) S = Z_{pz} / Z_{ez} = 1.118

Mean thickness of flange = (500 * 40 + 200 * 18 * 2) / 500 = 54.4 mm (Table XX gives

mean thickness of flange = 55 mm) Mean thickness > 40 mm

f_y = 230 MPa (Table 1 - IS: 800 - 2007, pp - 14)

Yield Moment, M_Y = f_Y Z_{ez} (Assuming f_y =250 MPa)

= 230 x 56782.9 x 10^3

= 13060.07 x 10^6 N mm

= 13060.07 kN m

Plastic Moment, M_p = f_Y Z_{pz}

= 230 x 63485.26 x 10^3

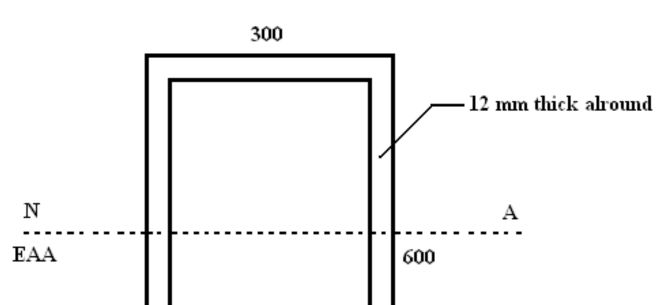
= 14601.61 N mm

= 14601.61 kN m

Working bending moment, M = M_p / 1.5 = 9734.41 kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

6. Box section, say 300 x 600 outer dimensions with 12 mm as uniform thickness



Consider the section as shown.

The beam is symmetrical about z - z axis and hence, NA and EAA co inside

$Z_{ez} = I_{zz} / y_{max}$

$= [300 * 600^3 / 12 - (300 - 12 * 2) * (600 - 12 * 2)^3 / 12] / (600 / 2)$

$= 3.35 \times 10^6$ mm^3

$Z_{pz} = (300 * 12 * 294 + 12 * 288 * 144 * 2) * 2$

$= 4.11 \times 10^6$ mm^3

(294 = 600 / 2 - 12 / 2 ; 288 = 600 / 2 - 12 ; 144 = 288 / 2) $S = Z_{pz} / Z_{ez} = 1.227$

$f_y = 250$ MPa (Table 1 - IS: 800 - 2007, pp - 14)

Yield Moment, $M_Y = f_Y Z_{ez}$ (Assuming $f_y = 250$ MPa)

$= 250 \times 3.35 \times 10^6$

$= 837.5 \times 10^6$ N mm

= 837.5 kN m

Plastic Moment, $M_p = f_Y Z_{pz}$

$= 250 \times 4.11 \times 10^6$

$= 1027.5 \times 10^6$ N mm

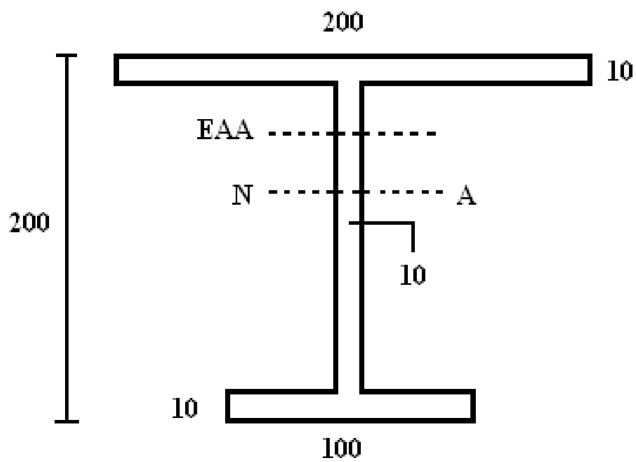
= 1027.5 kN m

Working bending moment, $M = M_p / 1.5 = 685$ kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

8. Unsymmetrical I section

Consider an Unsymmetrical c/s as shown



The section is unsymmetrical about the NA and hence the EAA has to be located. The NA has to be calculated as section properties are not available in SP (6)

Let y_t be the distance of NA from the top

y_t = [200 * 10 * 10 / 2 + 10 * 180 * (180 / 2 + 10) + 100 * 10 * (200 - 10 / 2)] /

Total area, A = (200 * 10 + 180 * 10 + 100 * 10)

= 80.21 mm

y_b = distance of NA from the bottom = 200 - 80.21 = 119.79 mm

I_{NA} = [200 * 10^3 / 12 + 200 * 10 * $(80.21 - 5)^2$] + [10 * 180^3 / 12 + 10 * 180 * $(100 - 80.21)^2$] + [100 * 10^3 /12 + 100 * 10 * $(195 - 80.21)^2$]

= 30.08 x 10^6 mm^4

$Z_{ez} = I_{NA} / y_{max}$ = 30.08 x 10^6 / 119.79 = 251.11 x 10^3 mm^3

Let the EAA be at distance y from the top

200 * 10 + 10 * (y - 10) = Total area / 2 = 4800 / 2 = 2400 y = 50 mm

Z_{pz} = 200 * 10 * 45 + 10 * 40 * 20 + 10 * 140 * 70 + 100 * 10 * 145

= 341 x 10^3 mm^3

S = Z_{pz} / Z_{ez} = 1.358

f_y = 250 MPa (Table 1 - IS: 800 - 2007, pp - 14)

Yield Moment, M_Y = f_Y Z_{ez} (Assuming f_y =250 MPa)

= 250 x 251.11 x 10^3

= 62.78 x 10^6 N mm

= 62.78 kN m

Plastic Moment, M_p = f_Y Z_{pz}

= 250 x 341 x 10^3

= 85.25 x 10^6 N mm

= 85.25 kN m

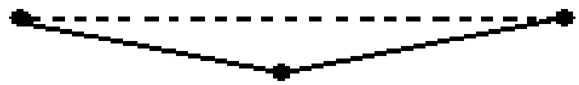
Working bending moment, M = M_p / 1.5 = 56.83 kN m

If UDL or other loads are required, maximum BM which depends on the type of loading and span is equated to working bending moment.

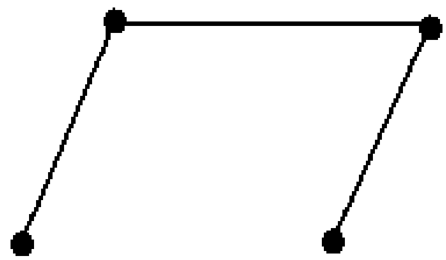
Mechanisms:

It can be defined as a situation in a structure in which sufficient number of plastic hinges are formed so as to cause collapse without further increase in the load carrying capacity. (collapse load) Mechanism breaks the structure into rigid links. When a mechanism is formed the structure is subjected to large rotations and deflections.

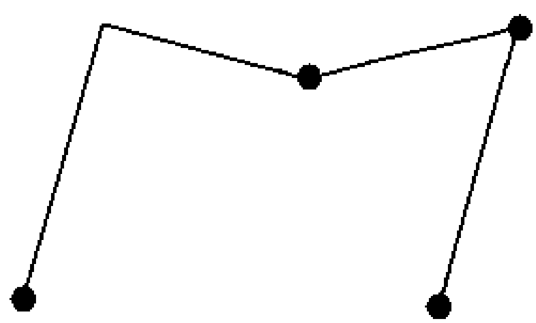
In an indeterminate structure, number of plastic hinges = SI makes the structure determinate and additional plastic hinge transforms the structure into a mechanism resulting in collapse. Mechanism results in instability of the structure with large deformations. The type of mechanism depends on the structure. Different types of mechanisms are indicated in the figures.



Beam Mechanism



Sway Mechanism



Combined Mechanism

4.7 Plastic Collapse load

It is the load corresponding to the collapse of a structure due to the formation of sufficient number of plastic hinges transferring the structure into a mechanism. It is also called ultimate load or limit load. The number of plastic hinges at collapse = SI +1, where SI = Static indeterminacy of the structure.

The collapse of a structure can be a partial, complete or over complete collapse depending on number of plastic hinges

In a structural system, if only a part of the structure fails, making the structure as a whole useless, then the collapse is said to be partial and the corresponding load is taken as the partial collapse load. Number of plastic hinges in a partially collapsed structure is

< SI +1. In rigid frames, if a beam fails, then it is called a partial collapse.

In a structural system, if complete collapse of the structure occurs, then the corresponding collapse load is taken as the true collapse load. Number of plastic hinges at collapse is = SI + 1. Usually, the value of the collapse load corresponds to any one mechanism which gives the lowest load.

In a structural system, if collapse of the structure occurs due to formation of more than the necessary plastic hinges , then the collapse is considered as over complete collapse and the corresponding load is said to be over complete collapse load.. Number of plastic hinges at collapse is > SI + 1. Two or more mechanisms give the same collapse load. This can happen in symmetrically loaded structures, where the plastic hinges do not lie on the axis of symmetry.

4.8 Conditions in Plastic Analysis:

In elastic analysis of structures, the conditions to be satisfied are : Equilibrium conditions - $\Sigma V = 0$; $\Sigma H = 0$ and $\Sigma M = 0$

Compatibility conditions - Continuity of slopes and deflections

BM < Yield Moment with suitable factor of safety

In plastic analysis of structures, the conditions to be satisfied are : Equilibrium conditions - $\Sigma V = 0$; $\Sigma H = 0$ and $\Sigma M = 0$

Mechanism condition or formation - Sufficient number of plastic hinges shall

be formed to transfer the structure into a mechanism

Plastic moment condition - BM at any point shall not exceed the plastic moment, M_p of the section ($M <$ or $= M_p$)

If all the three conditions are satisfied in plastic analysis, then we get the true collapse load. However, if any two conditions are satisfied, we get a load which is either below or above the true collapse load.

4.9 Theorems in Plastic analysis:

Based on the number of conditions satisfied, we have three theorems in plastic analysis.

Static Theorem (Lower bound theorem):

This theorem states that the load computed from any distribution of BMDs in equilibrium with external loads (safe and statically admissible BMD) so that the maximum BM in any member shall not exceed its plastic moment, M_p ($M <$ or $= M_p$) is less than or equal to the true collapse load. This theorem leads to equilibrium or static method of plastic analysis.

This theorem also called the safe theorem satisfies equilibrium and plastic moment condition.

Kinematic Theorem (Upper bound theorem):

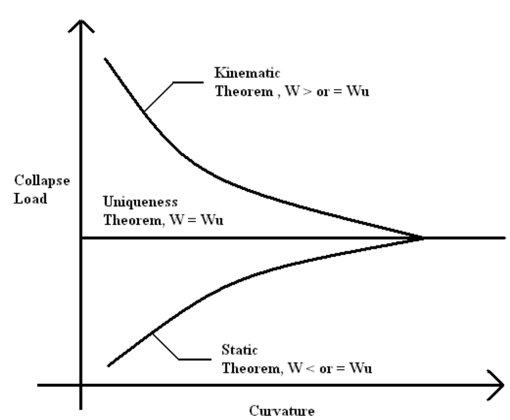
This theorem states that the load computed from any assumed kinematically admissible mechanism is greater than or equal to the true collapse load. This theorem leads to kinematic or mechanism method of analysis.

This theorem also called the unsafe theorem satisfies equilibrium and mechanism condition. A kinematically admissible mechanism (deformation) is one in which the deformation (rotation and deflection) under the load and supports satisfies the virtual work equation. (internal work by the plastic moment at plastic hinges = external work by the loads on the structure)

Uniqueness Theorem:

This theorem states that if the load evaluated by static and kinematic theorems is same, then it is the true collapse load. All the three conditions of plastic analysis are satisfied. According to this theorem, there is only one unique solution for a given structure, while there are innumerable possible solutions with other theorems.

All the three theorems are graphically shown in the figure below.



Methods of Plastic Analysis:

Equilibrium or Static Method:

This method is based on the lower bound theorem and consists of the following steps:

i) The redundant forces are chosen.

ii) Free BMD (Simply supported) of the structure is drawn. iii) Redundant BMD of the structure is drawn.

iv) The two BMDs are combined and the peak moments are determined. The peak

moment points are set the corresponding plastic moment, M_p. Also, check the number of plastic hinges formed.

v) The collapse load is determined using the equilibrium condition by equating the maximum simply supported BM to the plastic moment at the corresponding point.

vi) A check is applied so that plastic moment at any point is not > or = M_p at that point.
vii) BMD and SFD are drawn if required.

Kinematic or Mechanism method:

This method is based on the upper bound theorem and consists of the following steps:

i) The possible points of plastic hinges (N) formation is located. They are invariably formed at points of maximum BM, Continuous supports, Fixed supports, rigid joints, etc.,

ii) The number of independent mechanisms and combined mechanisms are determined.

In beams, only independent mechanisms are required. No. of independent mechanisms = N - SI , where N = No. of plastic hinges possible and SI = Static indeterminacy

iii) Using the principle of virtual work, the equilibrium equation involving the internal work done by plastic moment, M_p and the external work done by loads are equated to determine the plastic collapse load.

iv) A check is applied so that plastic moment at any point is not > or = M_p at that point.
v) BMD and SFD are drawn if required.

In this method , the elastic strain energy is ignored and the internal work done at only plastic hinges is considered.

Virtual Work Principle:

This principle is used for determining the collapse loads using mechanism method. This principle establishes the equilibrium relation between the work done by external loads and the internal work done by the plastic moment at specified location of plastic hinges. The principle is stated as follows:

If a structure is subjected to a set of loads which is in equilibrium, a virtual displacement (deflection or rotation) at specified points will not alter the equilibrium of the structure with total work done being zero. With this, the virtual work equation can be written as -

External Work done by the loads = Internal Work done by the plastic moment the location of the plastic hinges.

Plastic analysis of beams:

Beams can be analyzed by equilibrium or mechanism method. Usually, beams are subjected to transverse loads and only beam mechanisms occur. In continuous beam, each span is converted into a mechanism separately and lowest collapse load or the highest plastic moment is taken as the true value. It will be assumed that if one span collapses, the structure fails.

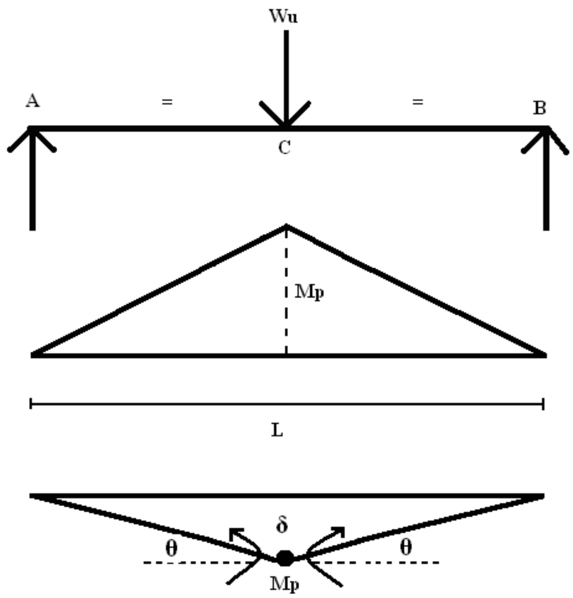
In beams, mechanisms can be relevant or irrelevant. Relevant mechanisms are the correct mechanisms, where plastic hinges are assumed at maximum BM locations such as fixed support, continuous support, concentrated loads, point of zero shear etc., Irrelevant mechanisms are incorrect mechanisms, where plastic hinges are assumed to be at incorrect locations. In analysis, only relevant mechanisms are considered.

No. of relevant mechanisms = N - SI N = No. of plastic hinges possible SI = Static indeterminacy

4.10 Problems on Plastic moment

Problem 1

Simply supported beam with central concentrated load -



Equilibrium method -

Maximum free BM = W_U * L / 4

Plastic moment of the section = M_P

Equating, W_U * L / 4 = M_P

W_U = 4 * M_P / L

Mechanism method -

Internal work done = 2 * M_P * θ External work done = W_U * δ From geometry, δ = (L / 2) * θ

Equating, 2 * M_P * θ = W_U * (L / 2) * θ

W_U = 4 * M_P / L

Problem 2

Propped Cantilever with Central concentrated load

Equilibrium method -

Maximum free BM = W_U * L / 4

Plastic moment of the section = M_P + M_P / 2 = 1.5 M_P

Equating, $W_U * L / 4 = 1.5 M_P$

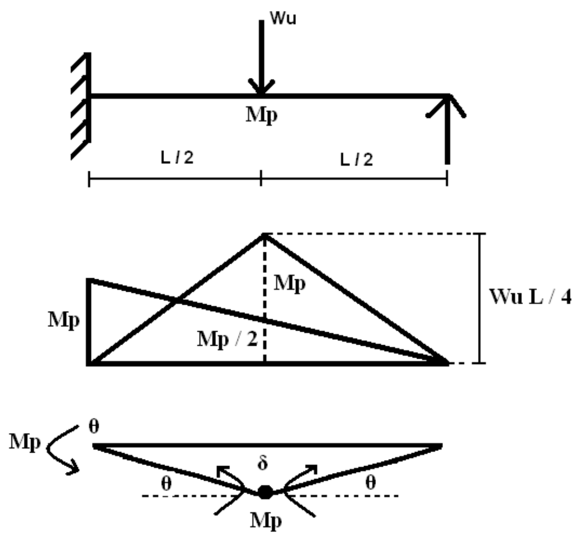
$W_U = 6 * M_P / L$

Mechanism method -

Internal work done = $3 * M_P * \theta$

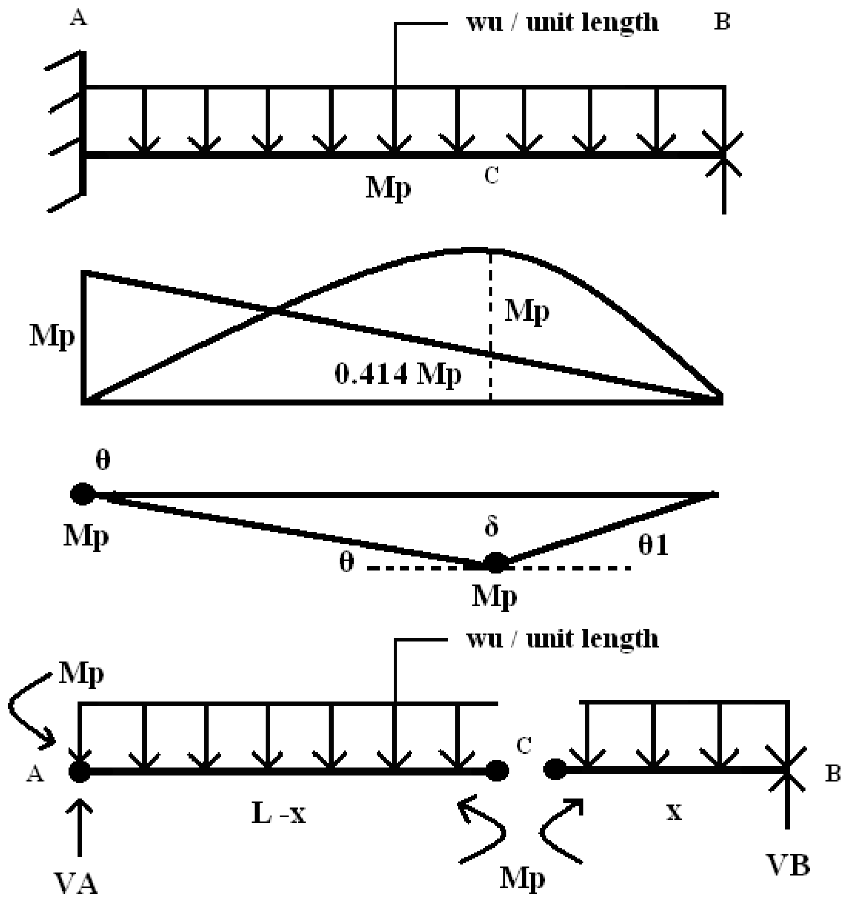
External work done = $W_U * \delta$

From geometry, $\delta = (L / 2) * \theta$ Equating, $3 * M_P * \theta = W_U * (L / 2) * \theta$ $W_U = 6 * M_P / L$



Problem 3

Propped Cantilever with UDL throughout



The location of the plastic hinge at C is carried out by segment equilibrium method. Let the hinge at C be at distance x from B.

BC = x and AC = (L- x) At C, SF = 0

Consider the segment BC,

$\Sigma M_B = 0$,

$M_P - w_u\ x^2 / 2 = 0$

$w_u = 2 M_P / x^2$ ------------------------------ (1)

Consider the segment AC,

$\Sigma M_A = 0$,

$w_u\ (L-x)^2\ / 2 - 2 M_p = 0$

$w_u = 4\ M_p / (L-x)^2$ --------------------------- (2)

Equating (1) and (2),

$2\ M_P / x^2 = 4\ M_p / (L-x)^2$

Simplifying, we get $x^2 + 2Lx - L^2 = 0$

$x = - L + \sqrt{2} * L$

$x = 0.414\ L$ (BC) and $L - x = 0.586\ L$ (AC)

$w_u = 11.66\ M_p / L^2$

Equilibrium method -

Simply supported BM at 0.414 L from B

$= w_u\ L / 2 * 0.414L - w_u * (0.414L)^2 / 2$

$= 0.1213\ w_u\ L^2$

Plastic moment of the section = $M_P + 0.414\ M_P = 1.414\ M_P$

Equating,

$1.414\ M_P = = 0.1213\ w_u\ L^2$

$w_u = 11.66\ M_p / L^2$

Mechanism method -

Internal work done = $2\ M_p\ \theta + M_p\ \theta_1$

External work done = $0.5 * L * \delta * w_u$ (Area under the mechanism * intensity of loading)

From geometry,

$\delta = 0.586L\ \theta = 0.414L\ \theta_1$

Equating,

$2\ M_p\ \theta + M_p\ \theta_1 = 0.5 * L * \delta * w_u$

$2 M_p \theta + M_p * 1.415 \theta = 0.5 * L * 0.586L \theta * w_u$

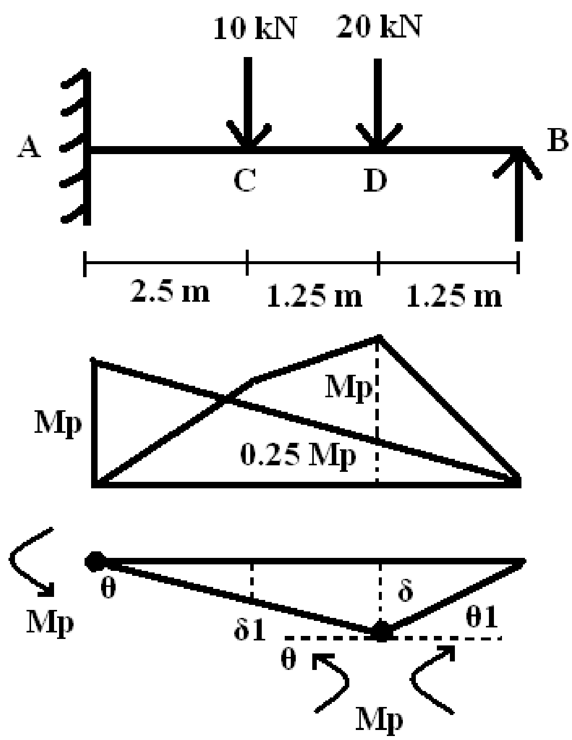
$3.415 M_p = 0.293 w_u L^2$

$w_u = 11.66 M_p / L^2$

SFD if required can be drawn either in terms of M_p or w_u

Problem 4

Propped Cantilever with two concentrated loads at collapse. Find M_p



Two plastic hinges can be formed at collapse. No. of possible plastic hinges = 3 (A, C and D) Two Relevant mechanisms are possible.

Equilibrium method - Plastic hinges at A and C

Maximum free BM = 10 * 2.5 = 25 kNM

Plastic moment of the section = $M_P + 0.5\ M_P = 1.5\ M_P$

Equating, $25 = 1.5\ M_P$

$M_P = 16.67$ kNm

Plastic hinges at A and D

Maximum free BM = 20 * 1.25 = 25 kNM

Plastic moment of the section = M_P + 0.25 M_P = 1.25 M_P

Equating, 25 = 1.25 M_P

M_P = 20 kNm

Adopt the largest value of M_P = 20 kNm for design. Mechanism method -

Internal work done = 2 * M_P * θ + M_P * θ_1

External work done = 20 * δ + 10 * δ_1

From geometry, $\delta = 3.75\ \theta$, $\delta = 1.25\ \theta_1$, $\theta_1 = 3\ \theta$ and $\delta_1 = 2.5\ \theta$

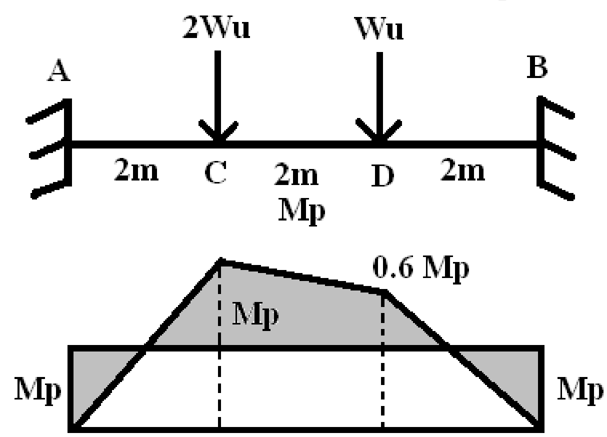
Equating, 2 * M_P * θ + M_P * θ_1 = 20 * δ + 10 * δ_1

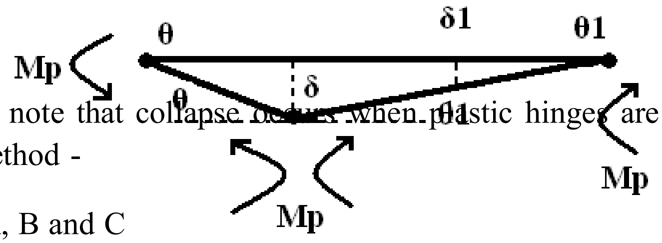
2 * M_P * θ + M_P * 3 θ = 20 * 3.75 θ + 10 * 2.5 θ

M_P = 20 kNm

Problem 5

Calculate the collapse load for the fixed beam shown with M_p = 100 kNm





By inspection, we note that collapse occurs when plastic hinges are formed at A, C and D. Equilibrium method -

Plastic hinges at A, B and C

Maximum free BM at C = (10 / 6) W_U * 2 = (10 / 3) W_U

Plastic moment of the section = M_P + M_P = 2 M_P

Equating, (10 / 3) W_U = 2 M_P = 200 kNm

W_U = 60 kN

Mechanism method -

Internal work done = 2 * M_P * θ + 2 * M_P * θ_1

External work done = 2 W_U * δ + W_U * δ_1

From geometry, δ = 2 θ, δ = 4 θ_1, θ_1 = 0.5 θ and δ_1 = 2 θ_1 = θ

Equating internal and external work done,

2 * M_P * θ + 2 * M_P * θ_1 = 2 W_U * δ + W_U * δ_1

2 * M_P * θ + 2 * M_P * 0.5 θ = 2 W_U * 2 θ + W_U * θ

3M_P = 5 W_U

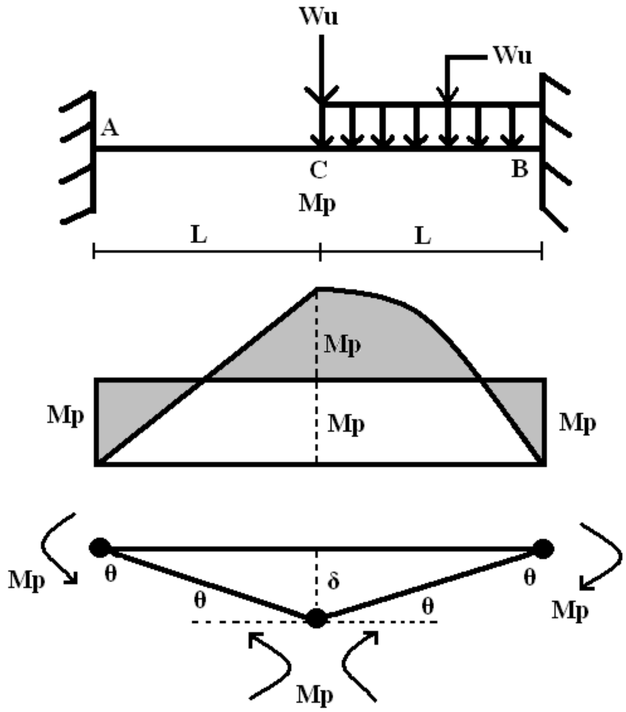
W_U = 0.6 M_P = 60 kN

The vertical reactions will be as in the case of simply supported beam.

Problem 6

Calculate the collapse load for the fixed beam shown.

The plastic hinges are formed at A , B and C based on crossing of shear force at C



The Simply supported reactions at A and B are $1.25W_U$ and $0.75W_U$ respectively.

Equilibrium method -

Plastic hinges at A, B and C

Maximum free BM at C = $0.75\ W_U * L$

Plastic moment of the section = $M_P + M_P = 2\ M_P$

Equating, $0.75\ W_U * L = 2\ M_P$

$W_U = (8 / 3)\ M_P / L$

Mechanism method -

Internal work done = $4 * M_P * \theta$

External work done = $W_U * \delta + 0.5 * L * \delta * W_U / L$

From geometry, $\delta = L\ \theta$

Equating internal and external work done,

$4 * M_P * \theta = W_U * \delta + 0.5 * L * L\theta * W_U / L$

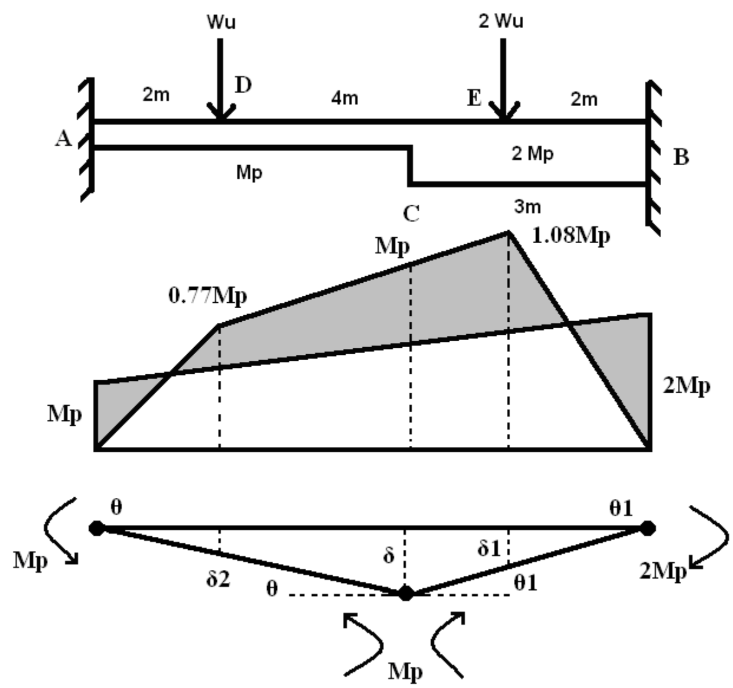
$4 * M_P * \theta = W_U * L\theta + W_U * L\theta / 2$

$4M_P = 1.5 W_U L$

$W_U = (8/3) M_P / L$

Problem 7

Fixed beam with different plastic moments. The beam is shown in the figure. The Simply supported reactions at A and B are $1.25W_U$ and $1.75W_U$ respectively.



Equilibrium method -

Plastic hinges at A, B and D

Maximum free BM at C = $1.25 W_U * 2 = 2.50 W_U$ Plastic moment of the section = $M_P + 1.25 M_P = 2.25 M_P$ Equating, $2.50 W_U = 2.25 M_P$

$W_U = 0.9 M_P$

Plastic hinges at A, B and C

Maximum free BM at C = $1.25 W_U * 5 - W_U * 3 = 3.25 W_U$ Plastic moment of the section = $M_P + 1.625 M_P = 2.625 M_P$ Equating, $3.25 W_U = 2.625 M_P$

$W_U = 0.8077\ M_P$

Plastic hinges at A, B and E

Maximum free BM at C = $1.75\ W_U * 2 = 3.5\ W_U$

Plastic moment of the section = $2\ M_P + 1.75\ M_P = 3.75\ M_P$

Equating, $3.5\ W_U = 3.75\ M_P$

$W_U = 1.0714\ M_P$

The lowest load of $W_U = 0.8077\ M_P$ is the true collapse load. Mechanism method -

Internal work done = $2 * M_P * \theta + 3 * M_P * \theta_1$

External work done = $W_U * \delta_2 + 2W_U * \delta_1$

From geometry, $\delta = 5\ \theta = 3\ \theta_1$, $\delta_1 = 2\ \theta_1, \delta_2 = 2\ \theta, \theta_1 = (5 / 3)\ \theta$

Equating internal and external work done,

$2 * M_P * \theta + 3 * M_P * \theta_1 = W_U * \delta_2 + 2W_U * \delta_1$

$2 * M_P * \theta + 3 * M_P * (5 / 3)\ \theta = W_U * 2\ \theta + 2W_U * 2 * (5 / 3)\ \theta$

$7M_P = (26 / 3)W_U$

$W_U = 0.8077\ M_P$

The support reactions in the fixed beam can be obtained as follows :

$2W_U * 6 + W_U * 2 + 2\ M_P - M_P - V_B * 8 = 0$

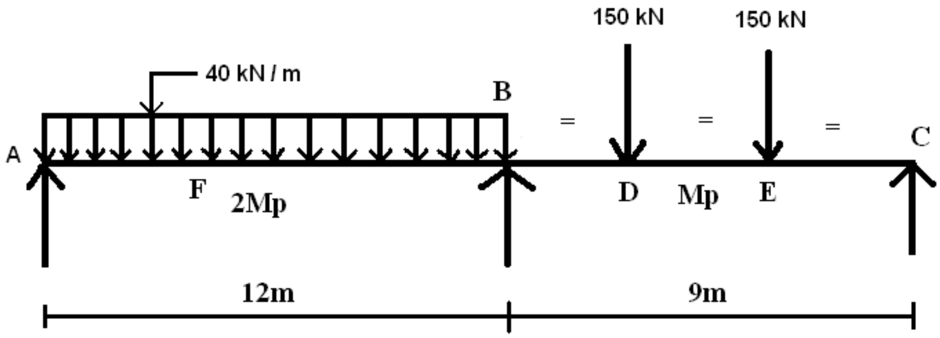
$V_B = 1.538\ M_P$

$V_A = 0.885\ M_P$

SFD can be drawn.

Problem 8

A two span continuous beam is shown below. All the loads are service loads.



No. of possible plastic hinges = 4 (B, Between A and B, under each concentrated load. No. of relevant mechanisms = 4 - 1 = 3

Each mechanism is similar to a propped cantilever. Mechanism 1 -

Plastic hinges will be at B and in between A and B. The distance x where the SF is zero is worked as in propped cantilever using segment equilibrium method. However, the value of plastic moment at B shall be chosen lesser of the two values ie M_P. The plastic moment in between A and B is $2M_P$.

The location of the plastic hinge at F is carried out by segment equilibrium method. Let the hinge at F be at distance x from A.

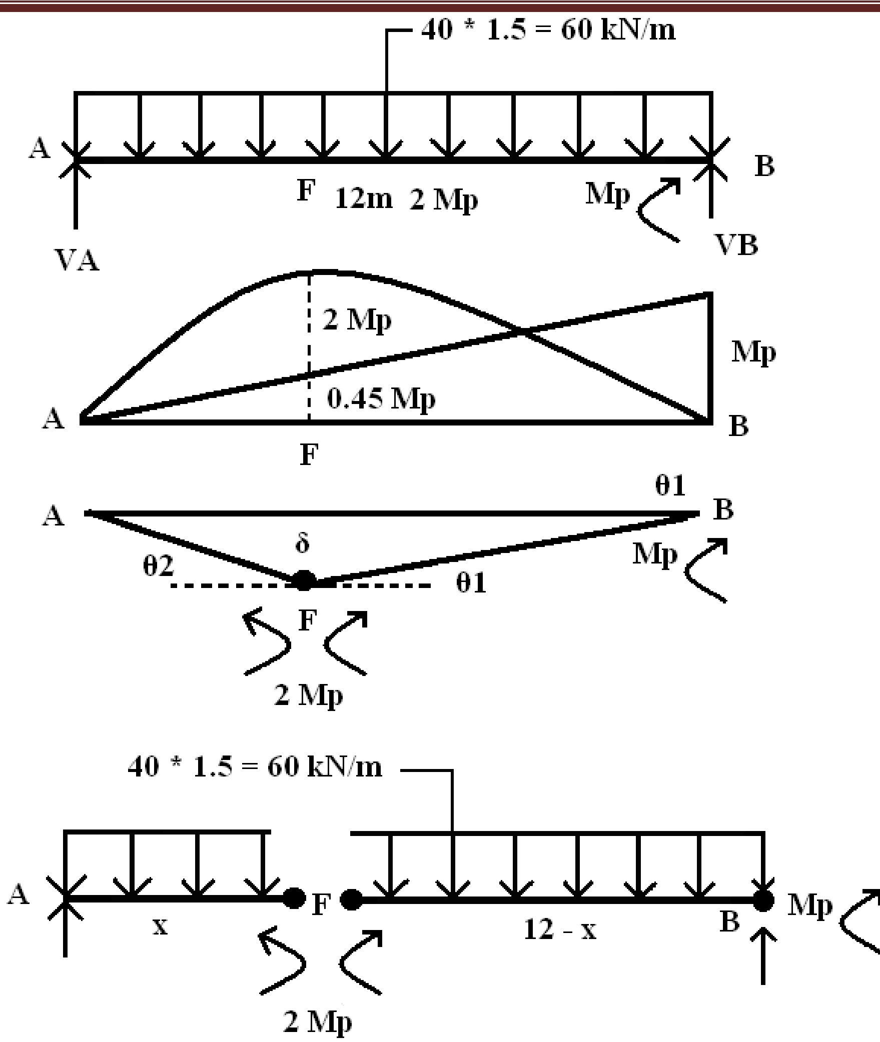
AF = x and BF = (12 - x) At F, SF = 0

Consider the segment AF,

$\Sigma M_A = 0$,

$2M_P - 60\, x^2 / 2 = 0$

$M_P = 15\, x^2$ ------------------------------ (1)



Consider the segment BF,

$\Sigma M_B = 0$,

$60\,(12 - x)^2\ / 2 - 3\ M_p = 0$

$M_P = 10\,(12 - x)^2$ -------------------------- (2)

Equating (1) and (2),

$15\ x^2 = 10\,(12 - x)^2$

$1.5\ x^2 = (12 - x)^2$

Simplifying, we get x = 5.394 m

$M_P = 436.43$ kNm

Equilibrium Method -

Simply supported BM at 5.394 m from A –

SS BM = 360 * 5.394 - 60 * 5.394^2 / 2 = 1068.98 kNm

Total Plastic Moment at the section = (2.0 + 0.45) M_P = 2.45 M_P

Equating SS BM to the total plastic moment,

1068.98 = 2.45 M_P

M_P = 436.32 kNm

Mechanism method -

Internal work done = 2 M_p θ_2 + 3M_p θ_1

External work done = 0.5 * 12 * δ * 60 = 360 δ

From geometry,

δ = 5.394 θ_2 = 6.606 θ_1

Equating,

2 M_p θ_2 + 3M_p θ_1 = 360 δ

2 M_p θ_2 + 3M_p * (5.394 / 6.606) * θ_2 = 360 * 5.394 θ_2

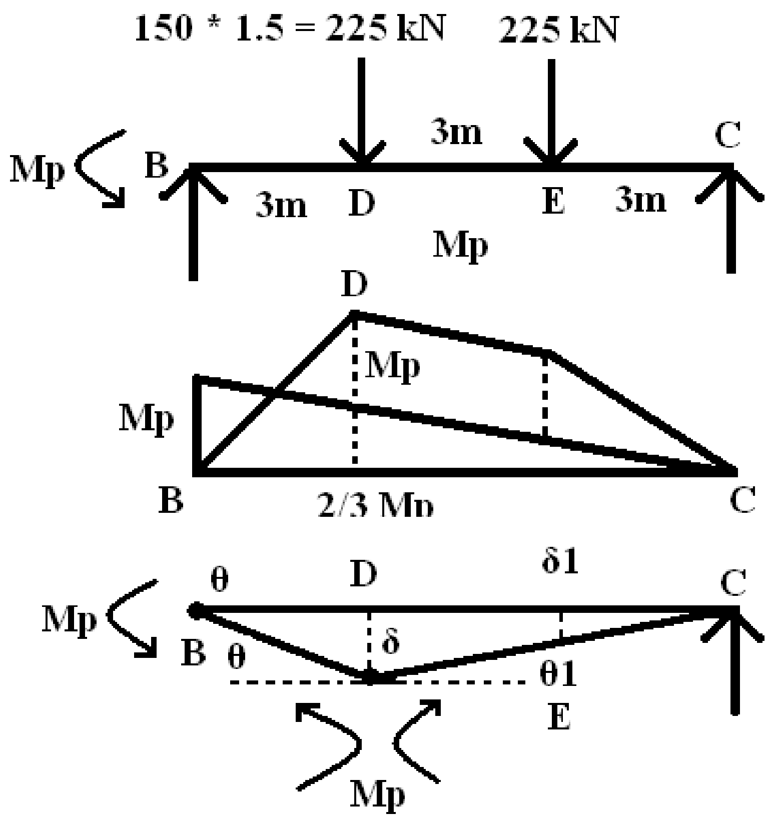
4.45 M_p = 1941.84

M_p = 436.37 kNm

Note : All methods give the same answer. The student shall check the question and accordingly answer. If no method is specified, then the student can have his /her choice.

Mechanism 2 -

Plastic hinges will be formed at B and D (under 150 kN load near B).



Equilibrium Method -

Simply supported BM at 3.0 m from B - SS BM = 225 * 3.0 = 675 kNm

Total Plastic Moment at the section = (1.0 + 2/3) MP = 5/3 MP

Equating SS BM to the total plastic moment,

675 = 5/3 MP

MP = 405 kNm

Mechanism method -

Internal work done = $2 M_p \theta + M_p \theta_1$

External work done = $225 * \delta + 225 * \delta_1$

From geometry,

$\delta = 3\theta = 6\theta_1$; $\theta_1 = 0.5\theta$ and $\delta_1 = 3\theta_1 = 1.5\theta$ Equating,

$2 M_p \theta + M_p \theta_1 = 225 * \delta + 225 * \delta_1$

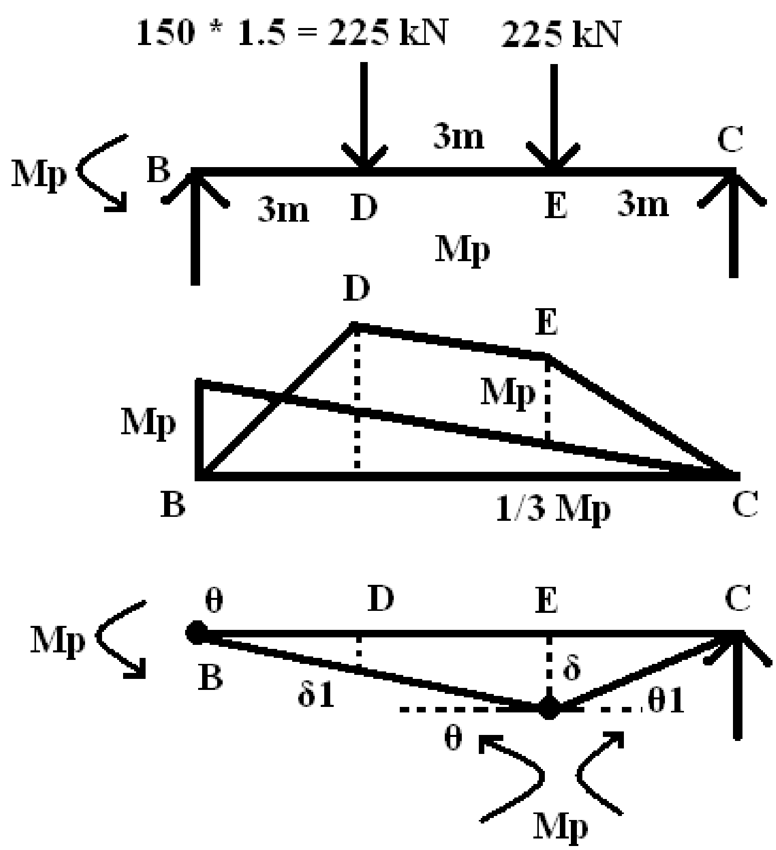
$2 M_p \theta + M_p * 0.5\theta = 225 * 3\theta + 225 * 1.5\theta$

$2.5 M_p = 1012.5$

$M_p = 405$ kNm

Mechanism 3 -

Plastic hinges will be at B and E (under 150 kN load near C)



Equilibrium Method -

Simply supported BM at 3.0 m from C - SS BM = 225 * 3.0 = 675 kNm

Total Plastic Moment at the section = (1.0 + 1/3) M_P = 4/3 M_P

Equating SS BM to the total plastic moment,

675 = 4/3 M_P

M_P = 506.25 kNm

Mechanism method -

Internal work done = $2 M_p \theta + M_p \theta_1$

External work done = $225 * \delta + 225 * \delta_1$

From geometry,

$\delta = 6 \theta = 3 \theta_1$; $\theta_1 = 2 \theta$

and $\delta_1 = 3 \theta$

Equating,

$2 M_p \theta + M_p \theta_1 = 225 * \delta + 225 * \delta_1$

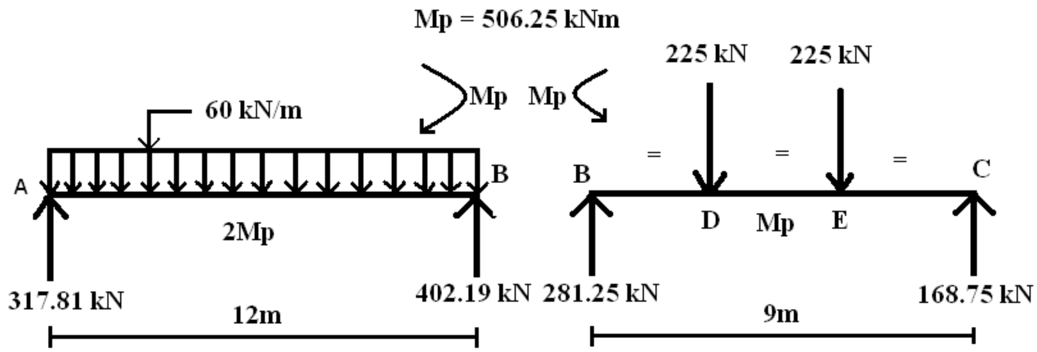
$2 M_p \theta + M_p * 2 \theta = 225 * 6 \theta + 225 * 3 \theta$

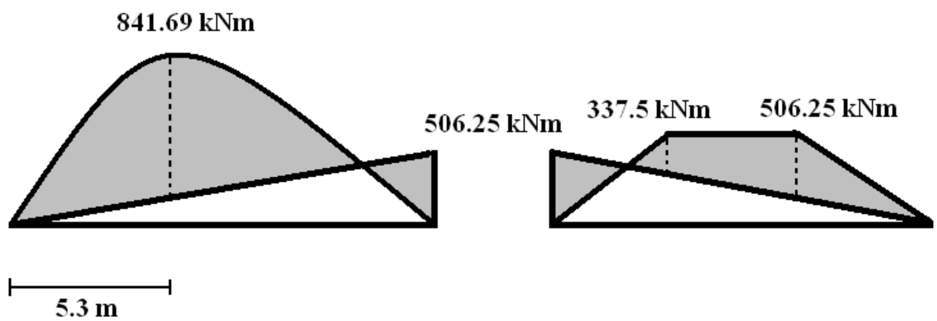
$4 M_p = 2025$

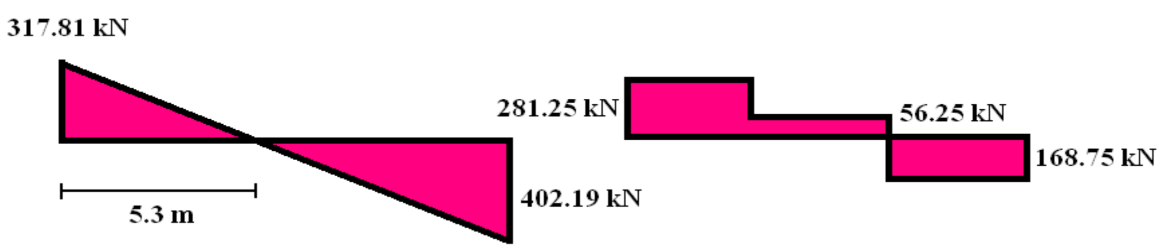
M_p = 506.25 kNm

The largest value of M_P shall be chosen. For the given problem, M_P = 506.25 kNm

The reactions for the beams are calculated as in elastic analysis by taking moments about any support and using the vertical equilibrium equation. The reactions are shown are below. BMD and SFD are also indicated. The BM has not exceeded the relevant plastic moment of the beam.







Note

The student shall work similar problems for practice with two and three span continuous beams with different support and loadings. If required BMD and SFD can be drawn. If end supports are fixed, plastic hinges are formed and the span behaves as fixed beam.

Sometimes, the value of Mp will be given for each span and the collapse load will be required to be calculated. In such problems, the collapse load is worked for each span and the lowest load is chosen as the true collapse load.

4.11 Assignment questions

1. What is plastic moment?

2. What is shape factor?
3. Explain plastic theory.
4. Explain plastic hinge concept.
5. Explain collapse load mechanism.

4.12 Outcome

Gives an idea about plastic moment

Gives an idea about plastic hinge

Gives knowledge about collapse load mechanism

4.13 Future Scope

By studying this concept one can analyze the frame sections which are subjected to plastic moments.